Students will be able to:
- identify the sample space of an experiment;
- apply set notation to simplify expressions involving events;
- apply Venn diagrams to the same.

- Toss of a coin. Sample space is $S = \{H, T\}$
The model for probabilities is $P(H) = 0.5$, $P(T) = 0.5$.
- Roll a dice. Sample space is $S = \{1, 2, 3, 4, 5, 6\}$
with $P(i) = \frac{1}{6}$ for $i = 1, 2, \ldots, 6$.
- Toss a quarter, a dime and a nickle together. Sample space is $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
Reasonable to model that the outcomes are equally likely.
So each outcome carries probability $\frac{1}{8}$.

Very important special case: Outcomes in $S$ are equally likely.
- In this case, for all $A$, $P(A) = \frac{|A|}{|S|}$
- Counting the elements in $A$ and $S$ becomes vitally important!
- Combinatorics: “Counting without counting”

Probability: Real-valued set function, $P$, satisfying:
Axiom 1: For all $A \subset S$, $P(A) \geq 0$ (Nonnegativity)
Axiom 2: If $S$ is the whole sample space then $P(S) = 1$
Axiom 3: Whenever $A_1, A_2, \ldots$ are mutually exclusive,

$$P \left( \bigcup_{i=1}^{\infty} A_i \right) = \sum_{i=1}^{\infty} P(A_i)$$ (Countable additivity)
Some theorems:

1. \( P(\emptyset) = 0. \)

2. If \( A_1, \ldots, A_n \) are mutually exclusive, then
   \[ P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) \]  
   (Finite additivity).

3. For any \( A \), \( P(A) = 1 - P(A^c) \).

4. If \( A \) implies \( B \) (i.e., \( A \subseteq B \)), then \( P(A) \leq P(B) \leq 1 \).

5. For any \( A \) and \( B \),
   \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

(5\*) For any \( A \) and \( B \) and \( C \),
\[
P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)
\]

(5\**) For any \( A_1, \ldots, A_n \),
\[
P\left( \bigcup_{i=1}^{n} A_i \right) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \cdots + (-1)^{n+1} P(A_1 \cap \cdots \cap A_n).
\]

### Example

A survey of a group’s viewing habits of gymnastics (G), baseball (B), and soccer (S) revealed that:
- 28% watched G
- 29% watched B
- 19% watched S
- 14% watched G and B
- 12% watched B and S
- 10% watched G and S
- 8% watched all three sports

What percentage of the group watched none of the three sports?

### Desired Outcomes

Students will be able to:
- understand equally likely outcomes;
- prove simple facts based on the three axioms of probability;
- use set notation or Venn diagrams for certain probability problems.