- Read Sections 4.5.1 and 4.7 (both editions)
- The next homework will not be posted until Monday; it will be due sometime the week of Feb. 6.

Notes: Also, I posted some more R code to serve as a companion to HW #2. I will continue to do this whenever it seems necessary, so please continue to ask R-related questions on email or in office hours so that I can tailor these supplementary materials in the future.

Jan. 27 4.3 Classification of States

If all of the states of a Markov chain communicate with each other, the chain is said to be

A. irreducible
B. ergodic
C. recurrent
D. transient

Notes: Correct answer: A. Obviously this is nothing but a terminology question.

Jan. 27 4.3 Classification of States

If \( P \) is the transition matrix for a Markov chain, what does the symbol \( P^n_{ij} \) mean?

A. \((P^n)_{ij}\)
B. \((P_{ij})^n\)
C. \(nP_{ij}\)
D. \(P_{ij}\)

Notes: Correct answer: A. C and D are silly answers; I only put them in because I needed 4 choices. The important thing to remember here is that the ambiguous notation \( P^n_{ij} \) refers to the \( i,j \)th entry of \( P^n \), which has a very meaningful interpretation for a Markov chain. On the other hand, \((P_{ij})^n\) does not have a useful interpretation.

Jan. 27 4.3 Classification of States

Gambler's ruin:

- Start with \( i \) dollars, \( 0 \leq i \leq N \).
- Bet 1 dollar per game (win $1 if you win, lose $1 if you lose).
- Continue until you have $0 or $N, then stay there.
- The communicating classes are \( \{0\} \), \( \{1, \ldots, N - 1\} \), \( \{N\} \).
- What is the probability that we wind up at \( N \) dollars (given \( X_0 = i \))?

Notes: We derived the answer to the final question using essentially the technique seen in Section 4.5.1 of the textbook. This is a very useful example, emphasizing the importance of conditioning and, occasionally, recurrence relationships when analyzing Markov chain behavior.
Theorem: Recurrence (or transience) is a class property. How do we prove this?

Notes: On the next slide are three equivalent ways to characterize recurrent states. We decided that \( \sum_{n=1}^{\infty} P_{ii}^n = \infty \) was the most useful of the three for this situation, and I asked you to try to prove that if \( \sum_{n=1}^{\infty} P_{ij}^n = \infty \) and there exist \( k \) and \( m \) with \( P_{ij}^k > 0 \) and \( P_{ji}^m > 0 \), then \( \sum_{n=1}^{\infty} P_{jj}^n = \infty \). Check Section 4.3 if you get stuck; we’ll wrap this up next Monday.

State \( i \) is recurrent if and only if, conditional on \( X_0 = i \),
- \( P(\text{ever revisiting } i) = 1 \)
- \( E(\#\{T > 0 : X_T = i\}) = \infty \).
- \( \sum_{n=1}^{\infty} P_{ii}^n = \infty \).