1. Let $N(t)$ be a non-homogeneous Poisson process with rate function $\lambda(t)$.

(a) Fix some $t > 0$. Derive the conditional distribution of the arrival times $S_1, \ldots, S_{N(t)}$, conditional on $N(t) = n$.

(b) As you did in Exercise 2(a) of Homework #5, use part (a) to suggest an algorithm for simulating a non-homogeneous Poisson process on $(0, t]$.

(c) Use your algorithm to simulate 10,000 realizations of a non-homogeneous Poisson process on $(0, 2]$ where $\lambda(t) = 24t^2$.

(d) Using the result of part (c), produce a histogram of the numbers of events in the interval $(0, 1]$. Add to your histogram the true theoretical values, and explain how you found them.

(e) Using the result of part (c), produce a histogram of the times until the first event. Add to your histogram the true theoretical density, and explain how you found it. What is the name of the true theoretical distribution?

2. Teams 1 and 2 are playing a game. The teams score points according to independent Poisson processes with rates $\lambda_1$ and $\lambda_2$, respectively. The game ends when one team has scored exactly $k$ points more than the other team. What is the probability that team 1 wins?

3. Consider $n$ components with independent lifetimes, where component $i$ has exponentially distributed lifetime with rate $\lambda_i$. Suppose that all components are initially in use and remain so until they fail.

(a) Find the probability that component 1 is the second component to fail.

(b) Find the expected time of the second failure.