1. A fair 6-sided die is rolled repeatedly. Let $X$ equal the number of rolls required to obtain the first 5 and $Y$ the number required to obtain the first 6. Calculate

(a) $E(X)$
(b) $E(X \mid Y = 1)$
(c) $E(X \mid Y = 5)$

2. Suppose that $X$ is exponentially distributed with parameter $\lambda$; i.e., $E(X) = 1/\lambda$. Calculate $P(X - 2 \leq t \mid X > 2)$ for an arbitrary $t > 0$. Based on your answer, what is the conditional distribution of $X - 2$ given $X > 2$?

3. A coin having probability $p$ of resulting in heads is successively flipped until the $r$th head appears. Let $X$ be the total number of flips required. Then $X$ is called a negative binomial random variable with parameters $r$ and $p$. (NB: A geometric random variable results in the special case $r = 1$.)

Argue that the probability mass function of $X$ is given by

$$p(k) = \binom{n-1}{r-1} p^r (1-p)^{n-r} \quad \text{for } k=r, r+1, \ldots.$$  

**Hint:** How many successes must there be in the first $n - 1$ trials?

4. Suppose that $X_1$ and $X_2$ are independent geometric random variables with the same parameter $p$. Prove that the conditional distribution of $X_1$ given $X_1 + X_2 = n + 1$, for some positive integer $n$, is discrete uniform.

**Hint:** The sum of independent geometric random variables with the same mean is negative binomial.

5. Suppose that you arrive at a party, along with a random number, $X$, of additional people, where $X \sim$ Poisson(10). The times at which people (including you) arrive at the party are independent uniform(0,1) random variables.

(a) Find the expected number of people who arrive before you.
(b) Find the variance of the number of people who arrive before you.

6. A short simulation exercise: Estimate the answers you obtained for Problem 5 above via simulation. You already have the answer so you can compare your estimates with the answer. Use 1000 replications of the process (one replicate of the process = one randomly sampled party.)

- First download and install R; see the course webpage, http://sites.stat.psu.edu/~dhunter/515/ for a link to “R statistical software links” provided by Dr. Haran.
- You can find a simple example for random variate simulation here: http://www.stat.psu.edu/~dhunter/515/hw/hw1ex.R. You can adapt this example to estimate the expectation and variance for this problem.

Ideally, you should also be reporting simulation (Monte Carlo) standard errors for your estimates; we will discuss this later in the course.

Because this is your first assignment and your R code here will be quite short, please include a printout of your R code with the assignment.

**Optional:** Typeset your homework solutions using \LaTeX. You might find it helpful to consult the “LaTeX mathematical writing links” at http://sites.stat.psu.edu/~dhunter/515/ (again, thanks to Dr. Haran!).