Question 1: According to the axiom of countable additivity for probability measures, if $E_1, E_2, E_3, \ldots$ is a sequence of disjoint events,

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \text{__________}. $$

Question 2: Suppose that $X$ is a Poisson random variable and $10P(X = 1) = P(X = 2)$. Find $E(X)$ and $\text{Var}(X)$.

Question 3: An insurance company believes that drivers can be divided into two classes, high-risk and low-risk. According to past data, a high-risk driver has an accident with probability 0.3 during a typical year. On the other hand, a low-risk driver has an accident with probability 0.1 during a typical year. Furthermore, 20% of policyholders are high-risk.

Suppose that a new policyholder has an accident in year one. What is the probability that the policyholder will have an accident in year two?

(You may assume that for a given individual, the occurrence of an accident in one year is independent of the occurrence of an accident in another year.)
Question 4: Suppose that $X$ is a random variable with moment generating function

$$M(t) = 0.7 + 0.3e^t.$$

Find the variance, $\text{Var}(X)$, and the fourth moment, $E(X^4)$.

Question 5: Suppose that you arrive at a party, along with a random number, $X$, of additional people, where $X \sim \text{Poisson}(10)$. The times at which people (including you) arrive at the party are independent uniform(0,1) random variables.

Find the mean and standard deviation of the number of people who arrive before you.

Question 6: If $X$ and $Y$ are iid standard normal random variables with joint density

$$c \exp\{- (x^2 + y^2)/2\} \text{ for } (x, y) \in \mathbb{R}^2,$$

demonstrate that $c = 1/(2\pi)$. Observe that this verifies the constant $1/\sqrt{2\pi}$ in the standard normal density,

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\{-x^2/2\}.$$