Stat 514 Solution 19 F. Xuan.

6.3.8. (a). $H_0$ $\theta = \theta_0$ vs $\theta \neq \theta_0$, $Y = \Sigma X_i$

$$L(\theta) = \frac{e^{-n\theta}\theta^{\Sigma X_i}}{\prod_{i=1}^{n}(X_i!)}$$

$$l(\theta) = -n\theta + (\Sigma X_i)\log\theta + \text{Constant}.$$

$$\Lambda = \frac{l(\theta_0)}{l(\hat\theta)} \Rightarrow -2\log\Lambda = -2\{-n(\theta_0-\hat\theta) + Y(\log\theta_0 - \log\hat\theta)\}$$

$$= 2n(\theta_0-\hat\theta) - 2Y(\log\theta_0 - \log\hat\theta).$$

where $\hat\theta = \bar{x} = \frac{Y}{n}$.

$$\Rightarrow -2\log\Lambda = 2(-Y + Y\log(Y/n) + \text{Constant}).$$

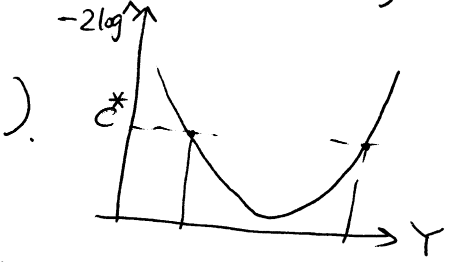
So $-2\log\Lambda$ is a function of $Y$.

One reject the null hypothesis iff $\Lambda \leq c$.

or equivalantly $-2\log\Lambda \geq c^*$.

Therefore the LR Test is based upon $Y = \Sigma X_i$

under the null hypothesis $Y \sim \text{Poisson}(n\theta_0)$




(b). $\theta_0 = 2$, $n = 5 \Rightarrow Y \sim \text{Poisson}(10)$.

$$\alpha = P(Y \leq 4) + P(Y \geq 17)$$

$\doteq 0.056$. (use poisson table or any software).

6.3.10. $\theta = \theta_0$ vs $\theta \neq \theta_0$.

$X_1 \cdots X_n \sim \Gamma(\alpha = 3, \beta = \theta)$.

$$L(\theta) = \frac{1}{2^n\theta^{3n}}\left(\prod_{i=1}^{n} X_i^2\right) e^{-\Sigma X_i/\theta}.$$

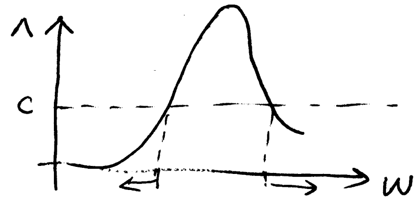
$$\Lambda = \frac{L(\theta_0)}{L(\hat{\theta})} = \left(\frac{\hat{\theta}}{\theta}\right)^{3n} \cdot \exp\left\{-\frac{\sum X_i}{\theta_0} + \frac{\sum X_i}{\hat{\theta}}\right\}$$

Notice that $\hat{\theta} = \frac{\sum X_i}{3n.} = \frac{W}{3n}$ where $W = \sum X_i$

$$\Lambda = W^{3n} \exp\left\{-\frac{W}{\theta_0}\right\} \cdot \text{Constant}.$$

Therefore LRT reject $H_0$ if $\Lambda \leq C$.

$$\Leftrightarrow W^{3n} \exp\left\{-\frac{W}{\theta_0}\right\} \leq C^*$$




So the LRT is based on $W$.

$$W = \sum X_i \sim \text{Gamma}(\alpha = 3n, \beta = \theta_0).$$

hence $\frac{2W}{\theta_0} \sim \chi^2(6n)$

(b). $P_{\theta_0}(W \leq C_1) + P_{\theta_0}(W \geq C_2) = 0.05$ $\quad \theta_0 = 3 \quad n = 5$

So. $W \sim \text{Gamma}(15, 3)$. or $\frac{2W}{\theta_0} \sim \chi^2(30)$

$\Rightarrow$ Since $\chi^2_{0.025, 30} = 16.791$ & $\chi^2_{0.975, 30} = 46.979$.

$\Rightarrow$ $C_1 = \frac{16.791\theta_0}{2} \doteq 25.1865$ $\quad C_2 \doteq 70.47$

6.3.16. (a). $H_0$ $\theta = 2$ vs $\theta \neq 2$

$-2\log\Lambda \xrightarrow{d}$ Chi-squre (1).

So reject $H_0$ if $-2\log\Lambda \geq C^* = \chi^2_{1,.95} = 3.841$