

Question 0.1 Let x_1, \dots, x_n be an iid sample from $N(0, \theta)$. the prior distribution of the parameter θ is Inverse Gamma with parameters a and b . This means

$$\frac{1}{\theta} \sim \text{Gamma}(a, b)$$

1. Derive the posterior distribution of $\Theta|X$.
2. Write down the posterior mean of $E(\Theta|X)$.
3. Is this posterior mean consistent for θ ? For whatever conclusion you get, Explain your result.

Question 0.2 Let x_1, \dots, x_n be an iid sample from $\text{Poisson}(\theta)$.

1. What is the sufficient statistics for θ ?
2. Is this poisson family complete?
3. Find the UMVUE of θ if it exists, otherwise show why it does not exist.
4. Find the UMVUE for $\delta = P(X = 0)$.
5. Derive the test statistic for

$$H_0 : \theta = 1 \quad \text{vs} \quad H_1 : \theta > 1$$

and write down the rejection region for a level α test.

6. Is this test Uniformly Most powerful? Explain your result.

Question 0.3 Let x_1, \dots, x_n be an iid sample from a continuous distribution $F(x)$ with density $f(x)$. Let Y_1, Y_2, \dots, Y_n be the order statistics for the x sample.

1. Derive the CDF or PDF for the random variable $W = F(X)$
2. Derive the PDF for the random variable $V = F(Y_2)$ where Y_2 is the second order statistics for x .
3. Now let z_1, z_2, \dots, z_n be the order statistics of a sample from the uniform $U(0, 1)$ distribution with size n . derive the PDF of z_2
4. What do you see from question 2 and question 3?