Review Problems for Midterm I
(Math/Stat 415, Section 1)

1. Suppose that $X_1, X_2, X_3$ and $X_4$ is a random sample of size $n = 4$ from the normal distribution $N(7.5, 9)$. Determine

(a) $P(\max_i X_i < 10.5)$ and $P(\max_i X_i \geq 4.5)$;
(b) $P(\min_i X_i \geq 4.5)$ and $P(\min_i X_i \leq 10.5)$;
(c) $E(X_1 X_2^2 (X_3 - 7.5)^2 e^{0.5X_4})$;
(d) $E(2X_1 + 3(X_2 - 7.5)^2 + e^{-X_4})$.

(e) Let $\bar{X}$ be the sample mean and $S^2$ the sample variance. Find the probability $P(|\bar{X} - 7.5| \leq 1.5, 0.648 < S^2 < 28.044)$.

2. Let $X_1, X_2$ and $X_3$ be mutually independent random variables with Poisson distributions having means 2, 1, 4, respectively.

(a) Find the moment-generating function of the sum $Y = X_1 + X_2 + X_3$.
(b) How is $Y$ distributed?
(c) Compute $P(3 \leq Y \leq 5)$.

3. The amount of time a baby naps has a mean of 61 minutes and a standard deviation of 16 minutes. The baby’s mother takes a random sample of times the baby naps for 100 days.

(a) Completely identify the distribution of sample mean. What allows you to use that distribution?
(b) A babysitter says that she will only take the job if the sample mean is at least 1 hour. What is the probability she will take the job?
(c) Another babysitter is 40% certain of taking the job, what would her cut off level be in terms of nap time?

4. Let $X$ and $Y$ equal the number of miles for compact cars and midsized cars, respectively, as reported in fuel economy ratings. Assume that $\mu_X = 24.5, \sigma_X = 3.8$ and $\mu_Y = 21.3, \sigma_Y = 2.7$. Let $\bar{X}$ and $\bar{Y}$ be the sample means of independent random samples of eight observations of $X$ and $Y$, respectively.

(a) What are the values of the means and variances of $\bar{X}$ and $\bar{Y}$?
(b) Assuming that $\bar{X}$ and $\bar{Y}$ are each (approximately) normally distributed, how is $\bar{X} - \bar{Y}$ distributed?
(c) Find the approximate probability $P(\bar{X} > \bar{Y})$.

5. Let $X$ equal the weight in grams of a miniature candy bar. Assume that $\mu = E(X) = 24.43$ and $\sigma^2 = \text{Var}(X) = 2.20$. Let $\bar{X}$ be the sample mean of a random sample of $n = 30$ candy bars. Find
(a) $E(\bar{X})$ and $\text{Var}(\bar{X})$;
(b) $P(24.17 \leq \bar{X} \leq 24.82)$, approximately.

6. A candy maker produces mints that have a label weight of 20.4 grams. Assume that the distribution of the weights of these mints is $N(21.37, 0.16)$.

(a) Let $X$ denote the weight of a single mint selected at random from the production line. Find $P(X > 22.07)$.
(b) Suppose that 16 mints are selected independently and weighted. Let $Y$ equal the number of these mints that weigh less than 20.857 grams. Then find the exact $P(Y \leq 2)$.
(c) Let $\bar{X}$ be the sample mean of the 16 mints selected in Part (b). Find $P(\bar{X} > 21.47)$.
(d) Suppose that 100 mints are selected independently and weighted. Let $W$ equal the number of these mints that weigh less than 20.857 grams. Find approximately $P(W \leq 5)$.

7. Let $X$ equal the number out of $n = 48$ mature aster seeds that will germinate when $p = 0.75$ is the probability that a particular seed germinates. Determine $P(35 \leq X \leq 40)$, approximately.

8. Let $X_1, X_2, \ldots, X_{30}$ be a random sample of size 30 from a Poisson distribution with a mean of $2/3$. Approximate

(a) $P(15 < \sum_{i=1}^{30} X_i \leq 22)$.
(b) $P(21 < \sum_{i=1}^{30} X_i < 27)$.

9. Let $T$ have a $t$-distribution with $r = 10$ degrees of freedom. Determine

(a) $P(T \geq 2.228)$.
(b) $P(T \leq -2.228)$.
(c) $P(|T| \geq 2.228)$.
(d) $P(-0.260 < T < 2.764)$.
(e) A constant $b$ so that $P(-b < T < b) = 0.95$.

10. Let $F$ have a distribution $F(3, 10)$. Determine

(a) $P(F \leq 3.71)$.
(b) $P(F > 6.55)$.
(c) $P(F \leq 1/14.42)$.
(d) $P(1/8.79 < F < 4.83)$. 

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