Notes on Whitworth Ch. 11: Disadvantage of Gambling

Fair bet: $E(\text{Profit}) = 0$
Even bet: Equal stakes by bettors
Notes that "fair bet" implies that play can continue indefinitely. If $\exists$ condition that play must stop under certain conditions then one player could be at a disadvantage.
A series of wagers, each being fair, may be collectively unfair if there is a contract to to stop at some stage.
Gambling: The act of exchanging something small and certain for something large and uncertain.
Advantage/disadvantage of a bet is related to the amt. of the bet relative to a man's total wealth (concept of utility?).
Absolute repetition: A man with £100 makes even bets of £1.
Relative repetition: A man with £100 makes even bets, staking only 1% of what he holds at each stage.

I. Absolute Repetition:
Eventually, the player will be bankrupted.

II. Relative Repetition:
The player's wealth will diminish over time.

The argument that gambling is a zero-sum game is fallacious, for the winner did not take into account the high chance of his being bankrupted when he started the game.

<u>Insurance is the reverse of gambling:</u> By buying insurance, we exchange a small but certain loss for a large contingent loss.

The Insurance Company, by pooling a large number of risks, and by charging a bit more than the math'l expectation of each risk as its premiums, can make a profit.

To the community, gambling is disadvantageous because it tends to oppose equable distn. of wealth. It tends to enrich the rich and make the poor poorer.

Consider a man with "available" funds of £n, interested in a speculation in which he has chances $p_1, p_2, p_3, \ldots$ of winning prizes worth $P_1, P_2, P_3, \ldots$ respectively. How much should he pay ~~for~~ a ticket in this game? Let X denote this amount, and assume that X is small compared to n.

Whitworth (Prop. LXXI, p. 241, 1965) shows that X satisfies the equation

$$\prod_{j=1}^{\infty}\left(1+\frac{P_j}{n}-\frac{X}{n}\right)^{p_j}=1.$$

Since X is small compared to n, we obtain

$$X=\frac{\left[\prod_{j=1}^{\infty}\left(1+\frac{P_j}{n}\right)^{p_j}\right]-1}{\sum_{j=1}^{\infty}\frac{p_j}{n+P_j}}.$$

Application to the St. Petersburg paradox: