CHAPTER 12
MORE ABOUT CONFIDENCE INTERVALS

How much difference is there between the mean pulse rates of women and men? What is the mean amount that Pen State students study per day? Both of these questions are examples of questions that involve the estimation of a population value. The statistical method we use to make estimates is a confidence interval. You may recall that in Chapter 10 we encountered this definition:

A confidence interval is a range of values that is likely to contain the population value.

In this Chapter, we’ll learn how to determine confidence intervals for several different types of research questions. The general format of the intervals will be the same in each scenario. For each scenario, the objective is also the same, which is that we use sample information to estimate a population value. The following two definitions will be useful in the discussion of this objective:

- A parameter is a population characteristic. The numerical value of a parameter is not known. We have to estimate the parameter using sample information.
- A statistic, or estimate, is a characteristic of a sample. A statistic estimates a parameter.

12. 1 Examples of Different Situations Involving Estimation

Here are four common research situations that can be analyzed using confidence intervals. We’ve already covered one of those situations, the estimation of a proportion, but we include it here to emphasize the fact that the confidence intervals presented in this Chapter all have the same general structure. For each situation, we provide two examples, the notation for the parameter, and notation for the sample estimate.

1. Estimating the proportion falling into a category of a categorical variable.

- Example Research Questions:
  - What proportion of Penn State students believe there is extraterrestrial life?
  - What proportion of American adults think marijuana should be legalized?
2. **Estimating the mean of a quantitative variable.**
   - **Example Research Questions:**
     - What is the mean time that Penn State students study per day?
     - What is the mean pulse rate of women?
   - **Notation for population parameter:** \( \mu \) (pronounced “mu”)
   - **Notation for Sample Estimate:** \( \bar{x} \), the sample mean.

3. **Estimating the difference between two populations with regard to the proportion falling into a category of a categorical variable.**
   - **Example Research Questions:**
     - How much difference is there between the proportions of PSU males and PSU females with regard to the proportion who believe there is extraterrestrial life.
     - How much difference is there between men who snore and men who don’t snore with regard to the proportion who have heart disease?
   - **Notation for population parameter:** \( p_1 - p_2 \) where \( p_1 \) and \( p_2 \) represent the proportions in populations 1 and 2 respectively.
   - **Notation for Sample Estimate:** \( \hat{p}_1 - \hat{p}_2 \), the difference between two sample proportions.

4. **Estimating the difference between two populations with regard to the mean of a quantitative variable.**
   - **Example Research Questions:**
     - How much difference is there between the mean foot lengths of men and women?
     - How much difference is there between science majors and liberal arts majors with regard to the mean hours of spent studying per week?
   - **Notation for population parameter:** \( \mu_1 - \mu_2 \) where \( \mu_1 \) and \( \mu_2 \) represent the means in populations 1 and 2 respectively.
   - **Notation for Sample Estimate:** \( \bar{x}_1 - \bar{x}_2 \), the difference between two sample means.
12. 2 Approximate 95% Confidence Intervals

It’s not very likely that a sample estimate will exactly equal the population parameter. There’s nearly always going to be an “error” due to the act of sampling. The size of this error varies from sample to sample, and because we don’t know the population value we don’t know the precise amount of error in any particular estimate. We can, however, determine approximately the “average” sampling error that may occur over the many possible samples that could be taken from a population.

**Standard Error of a Sample Statistic**

The **standard error** of a sample statistic measures, roughly, the average difference between the statistic and the population parameter. This “average difference” is over all possible random samples that can be taken from the population.

With a sufficient sample size, it is generally true that for 95% of all random samples from a population, the difference between the sample statistic and the population parameter is less than two standard errors. The phrase “sufficient sample size” is, of course, a vague phrase. Also, the sample size requirements depend upon whether you’re estimating a proportion or a mean. We’ll provide guidelines within the context of each confidence interval described in this section.

In Chapter 10, we learned that an approximate 95% confidence interval for a proportion is calculated as $\text{sample proportion} \pm 2 \times \text{standard error}$. The general format of this interval applies to all four situations described in this Chapter.
Calculating the Standard Error

The general format of the approximate 95% confidence interval is the same for all of the situations in this Chapter. The formula for calculating of the standard error, however, depends upon the situation. Here are the formulas for the standard error in the four different scenarios we’re considering. We use the notation \text{s.e.}(\text{estimate}) to represent a standard error. Remember that in all situations, the standard error measures the “average” difference between the sample estimate and the population parameter.

1. **Standard Error of a Sample Proportion**

\[
\text{s.e.}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]

2. **Standard Error of a Sample Mean**

\[
\text{s.e.}(\bar{x}) = \frac{s}{\sqrt{n}}
\]

3. **Standard Error of the Difference between Two Proportions**

\[
\text{s.e.}(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}
\]

4. **Standard Error for the Difference between Two Means**

\[
\text{s.e.}(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}
\]
Example 1 Mean Hours per Day that College Students Watch Television

In one of our class surveys, a question was “In a typical day, about how much time do you spend watching television? Immediately below, there’s a numerical summary of the responses (in hours). The sample mean was 2.09 hours for the n=175 students who participated in the survey.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>TrMean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV</td>
<td>175</td>
<td>2.09</td>
<td>2.000</td>
<td>1.950</td>
<td>1.644</td>
<td>0.124</td>
</tr>
</tbody>
</table>

Based on this information, what is our confidence interval estimate of the mean hours of television per day in the population represented by this sample? From the Minitab output, we see that the standard error of the mean is 0.124 (look under “SE Mean”). We can verify this value by computing

\[
\text{s.e.}(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{1.644}{\sqrt{175}} = 0.124
\]

A somewhat loose interpretation of this standard error is that it measures, roughly, the likely difference between the sample mean and unknown value of the population mean.

An approximate 95% confidence interval estimate of the population mean is

\[
\bar{x} \pm 2 \times \text{s.e.}(\bar{x})
\]

\[
2.09 \pm 2 \times 0.124
\]

\[
2.09 \pm 0.248
\]

\[
1.842 \text{ to } 2.338
\]

This interval is likely to contain the population mean. In a research article, the confidence interval might be described as follows: We are 95% confident that the mean time that college students spend watching television per day is somewhere between 1.842 and 2.338 hours.
Turn On Your Mind

What population do you think is represented by the sample of 175 students in Example 1?

Does the confidence interval in Example 1 tell us that 95% of the students watch television between 1.842 and 2.338 hours of television per day? If not, what exactly does the interval tell us? (Figure 12.1, a boxplot of the individual responses should help you with this task.)

Figure 12.1 Boxplot of Hours per Day of Television

Example 2 Do Men Lose More Weight by Diet or by Exercise?

Wood et al. (1988), also reported by Iman (1994, p. 258), studied a group of 89 sedentary men for a year. Forty-two men were placed on a diet, while the remaining 47 were put on an exercise routine. The group on a diet lost an average of 7.2 kg, with a standard deviation of 3.7 kg. The men who exercised lost an average of 4.0 kg, with a standard deviation of 3.9 kg (Wood et al., 1988). The difference between the two means is 

\[ \bar{x}_1 - \bar{x}_2 = 7.2 - 4.0 = 3.2 \text{ kg} \]  

(about 7 pounds) with the dieters losing more. The standard error of this statistic is 

\[ \text{s.e.}(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{42} + \frac{s_2^2}{49}} = \sqrt{\frac{3.7^2}{42} + \frac{3.9^2}{49}} = 0.81 \]
The standard error measures the accuracy of the statistic \( \bar{x}_1 - \bar{x}_2 \). An approximate 95% confidence interval for the difference between population means is
\[
sample \text{ difference } \pm 2 \times \text{standard error}
\]
\[
3.2 \pm 2 \times 0.81
\]
\[
3.2 \pm 1.62
\]
It is likely that the difference between the mean losses is contained in this interval.
Again, we note that the dieters lost more weight.

12.3 General Confidence Intervals for Means

In the previous section, we looked at approximate 95% confidence intervals. The format, if the sample was large, was \( sample \text{ estimate } \pm 2 \times \text{standard error} \). This format is a specific case of a more generally useful format for a confidence interval:
\[
sample \text{ estimate } \pm \text{multiplier } \times \text{standard error}.
\]
In general, the appropriate multiplier depends upon the desired confidence level, the sample size, and the type of statistic that we’re dealing with. When we’re dealing with means, the multiplier is denoted as \( t^* \) and the general format of a confidence interval confidence interval for a population mean is:
\[
sample \text{ mean } \pm t^* \times \text{standard error}.
\]
The multiplier, \( t^* \), is determined using a probability distribution called the \( t \)-distribution. This is a close cousin to the standard normal distribution. Theoretically, the \( t \)-distribution arises when a sample standard deviation is used in place of a population standard deviation when a standardized score for the observed mean is calculated. A \( t \)-distribution has a bell shape, it’s centered at 0, and it is more spread out than the standard normal curve in that there’s more probability in the extreme areas than there is for the standard normal curve.

A parameter called \textit{degrees of freedom}, abbreviated as \( df \), is associated with any \( t \)-distribution. In most applications, this parameter is a function of the sample size for the problem, but the formula for the degrees of freedom depends on the type of problem. For problems involving inference about a single mean, \( df = n-1 \) where \( n \) is the sample
size. A “calculus” property of the t-distribution is that as the degrees of freedom value increases, the distribution gets closer to the standard normal curve.

All of this may sound confusing, but it’s actually simple to determine a 95% confidence interval for a population mean. All we need is a table (or statistical software or the right calculator) to find the multiplier for the desired confidence level. Table 12.1 shows multipliers for four different confidence levels.

**Example 3** Imagine that we are calculating a 95% confidence interval estimate of a mean based on a sample of n=9 values. The degrees of freedom=9-1=8. From table 12.1, we see that the correct multiplier is 2.31. So the 95% confidence interval will be calculated as \( \bar{x} \pm 2.31 \times \frac{s}{\sqrt{n}} \).

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### Calculating a Confidence Interval for a Population Mean

1. Determine the sample mean and standard deviation (\( \bar{x} \) and s)
2. Calculate the standard error of the mean. \( s.e.(\bar{x}) = \frac{s}{\sqrt{n}} \).
3. Calculate \( df = n-1 \) and choose a confidence level. Use Table 12.1 (or statistical software) to find \( t^* \).
4. The interval is \( \bar{x} \pm t^* \times s.e.(\bar{x}) \)

**Note 1**: This procedure is correct for all sample sizes, but it is common “conceptual” practice to use the value 2 as the multiplier when \( n \geq 30 \).

**Note 2**: In Minitab, the calculations are automated. Click Stat>Basic Stats>1 Sample t
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<td>1.65</td>
<td>1.96</td>
<td>2.33</td>
<td>2.58</td>
</tr>
</tbody>
</table>
Example 4. Students in a statistics class at Penn State were asked, “About how many minutes do you typically exercise in a week.” The responses from 16 women in the class were: 60, 240, 0, 360, 450, 200, 100, 70, 240, 0, 60, 360, 180, 300, 0, 270. Here is output from the Minitab procedure for determining a 95% confidence interval for the population mean.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>95.0 % CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercise</td>
<td>16</td>
<td>180.6</td>
<td>144.3</td>
<td>36.1</td>
<td>(103.7, 257.5)</td>
</tr>
</tbody>
</table>

The sample mean is 180.6 minutes of exercise per week and we are 95% confident that the population mean is captured by the interval 103.7 minutes to 257.5 minutes. The standard error is \( \frac{s}{\sqrt{n}} = \frac{144.3}{\sqrt{36}} = 36.1 \). The \( df=n-1=16-1=15 \). In Table 12.1, we see that, for this problem, the multiplier for a 95% confidence interval is 2.13. The 95% confidence interval was calculated as 180.6 ± 2.13 × 36.1.
The Difference between Two Means

Unfortunately, there’s a muddy and confusing mathematical story behind the calculation of a confidence interval for the difference between two population means. On the surface, however, the story is easy.

An approximate confidence interval for $\mu_1 - \mu_2$ is $\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{s_1^2/n_1 + s_2^2/n_2}.$

The problem, however, is that it’s not exactly mathematically correct to use a t-distribution to determine the multiplier. It is approximately correct to do so, but the approximation involves an extraordinarily ugly formula for the degrees of freedom.

\[
\text{Approximate degrees of freedom used to find } t^* \\
\text{Confidence Interval for } \mu_1 - \mu_2 \\
df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}.
\]

The good news is that we’ll let Minitab do the work for us, and you won’t have to calculate these degrees of freedom “by hand.” Also, remember that for a sufficiently large sample size we can approximate a 95% interval by using the value “2” as the multiplier.

Example 5. The Effect of a Stare on Driving Behavior

In a lecture earlier this semester, we discussed an experiment done by social psychologists at the University of California at Berkeley. The researchers either did not stare or did stare at automobile drives stopped at a campus stop sign. The researchers then measured how long it took the drivers to drive from the stop sign to a mark on the other side of the intersection. The researchers believed that the average crossing times would be faster for those who experience the stare. The crossing times, in seconds, were:

**No Stare Group (n=14):** 8.3, 5.5, 6.0, 8.1, 8.8, 7.5, 7.8, 7.1, 5.7, 6.5, 4.7, 6.9, 5.2, 4.7

**Stare Group (n=13):** 5.6, 5.0, 5.7, 6.3, 6.5, 5.8, 4.5, 6.1, 4.8, 4.9, 4.5, 7.2, 5.8
Here’s Minitab output for a 95% confidence interval for the difference between population means.

Two sample T for CrossTime

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>NoStare</td>
<td>14</td>
<td>6.63</td>
<td>1.36</td>
<td>0.36</td>
</tr>
<tr>
<td>Stare</td>
<td>13</td>
<td>5.59</td>
<td>0.822</td>
<td>0.23</td>
</tr>
</tbody>
</table>

95% CI for \( \mu_{\text{NoStare}} - \mu_{\text{Stare}} \): (0.14, 1.93)
T-Test \( \mu_{\text{NoStare}} = \mu_{\text{Stare}} \) (vs >): \( T = 2.41 \) \( P = 0.013 \) \( DF = 21 \)

For the “No Stare” data, the sample mean is \( \bar{x}_1 = 6.63 \) seconds, while for the “Stare” data, \( \bar{x}_2 = 5.59 \) seconds. The difference between the sample means is 6.63 - 5.59 = 1.04 seconds. The 95% confidence interval for \( \mu_1 - \mu_2 \) is 0.14 to 1.93 seconds. This interval is likely to contain the true population difference between the means.

We see from the output (the final item) that \( df = 21 \) for this problem. Minitab calculated this quantity using the approximation formula on the previous page. From Table 12.1, we see that when \( df = 21 \), the \( t^* \) multiplier for a 95% confidence interval is 2.08. So, Minitab calculated the confidence interval as

\[
\bar{x}_1 - \bar{x}_2 \pm \text{multiplier} \times \text{standard error}
\]

\[
= 1.04 \pm 2.08 \times \sqrt{ \frac{1.36^2}{14} + \frac{0.822^2}{13} }
\]

**A Special Case: Assuming Equal Variances**

If we assume that the two populations have the same standard deviations, a “clean” mathematical solution for a confidence interval for the difference between two population means occurs. With this assumption, the solution involves using a quantity called the *pooled variance* in place of each of \( s_1^2 \) and \( s_2^2 \) in the formula for the standard error of the
difference between the sample means. This causes the degrees of freedom for the t multiplier to be exactly \( df = n_1 + n_2 - 2 \).

The pooled variance is computed as:

\[
s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}
\]

Substituting the pooled variance for each of \( s_1^2 \) and \( s_2^2 \) in the standard error formula produces this formula:

\[
s.e.(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}} = \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}
\]

So, the confidence interval for \( \mu_1 - \mu_2 \), the difference between the population means is

\[
\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}
\]

where \( t^* \) is found using \( df = n_1 + n_2 - 2 \).

The good news is that Minitab, or any other statistical software program, will do the calculations for us. In Minitab, check the dialog box item that says “Assume Equal Variances.”

**Example 5 Continued**

Here’s the Minitab output that resulted from clicking “Assume Equal Variance

Two sample T for CrossTime

<table>
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<tr>
<th>Group</th>
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<td>0.82</td>
<td>0.23</td>
</tr>
</tbody>
</table>

95% CI for \( \mu_1 \) (NoStare) - \( \mu_2 \) (Stare): \( 0.14, 1.94 \)

T-Test \( \mu_1 \) (NoStare) = \( \mu_2 \) (Stare) (vs >): \( T = 2.37 \) \( P = 0.013 \) \( DF = 25 \)

Both use Pooled StDev = 1.14

The 95% confidence interval for \( \mu_1 - \mu_2 \) is 0.14 to 1.94 seconds, an interval that’s only slightly different than the interval that we got when we didn’t assume equal variances.

Note that the degrees of freedom are reported as 25. The calculation is \( df = n_1 + n_2 - 2 = 14 + 13 - 2 \). Also, note that a “pooled standard deviation” is reported.

This is the square root of the “pooled variance” described above.
Summary of Formulas for Confidence Intervals

The basic structure of intervals for the parameters in the table below is

\[ \text{sample statistic} \pm \text{multiplier} \times \text{standard error}. \]

The following table describes the specific details for each type of parameter we’ve considered. Remember that for a sufficiently large sample size, the multiplier for a 95% confidence interval is approximately equal to 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Statistic</th>
<th>Standard Error</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Mean</td>
<td>( \mu )</td>
<td>( \bar{x} ) ( \frac{s}{\sqrt{n}} )</td>
<td>( t^* ) (see 1)</td>
</tr>
<tr>
<td>Difference Between Means</td>
<td>( \mu_1 - \mu_2 )</td>
<td>( \bar{x}_1 - \bar{x}_2 ) ( \sqrt{ \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} } )</td>
<td>( t^* ) (see 2)</td>
</tr>
<tr>
<td>One proportion</td>
<td>( p )</td>
<td>( \hat{p} ) ( \sqrt{ \frac{\hat{p}(1-\hat{p})}{n} } )</td>
<td>( z^* ) (see 3)</td>
</tr>
<tr>
<td>Difference Between Proportions</td>
<td>( p_1 - p_2 )</td>
<td>( \hat{p}_1 - \hat{p}_2 ) ( \sqrt{ \frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2} } )</td>
<td>( z^* ) (see 3)</td>
</tr>
</tbody>
</table>

(1) Use Table 12.1, \( \text{df} = n - 1 \)

(2) Use Table 12.1, \( \text{df} = \frac{\left( \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{1}{n_1-1} \left( \frac{s_1^2}{n_1} \right)^2 + \frac{1}{n_2-1} \left( \frac{s_2^2}{n_2} \right)^2} \)

(3) \( Z^* \) multipliers are:

<table>
<thead>
<tr>
<th>Conf. Level</th>
<th>( Z^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>1.65</td>
</tr>
<tr>
<td>0.95</td>
<td>1.96, usually rounded to 2</td>
</tr>
<tr>
<td>0.98</td>
<td>2.33</td>
</tr>
<tr>
<td>0.99</td>
<td>2.58</td>
</tr>
</tbody>
</table>