**Stat/Math 418: Practice for Midterm**

**Problem 1**
How many letter strings of length 4 can be produced from \{A,B,C,D,E,F,G\} if:
1. The string must contain at least two A’s.
2. The string must contain exactly two A’s.

**Problem 2**
Urn 1 and Urn 2 each contain two balls. Balls can be red or white. One ball is selected at random from Urn 1, and one ball is selected at random from Urn 2:
1. Write down the sample space for the experiment.
2. Write down the probabilities for single outcomes in the two cases
   a. Urn 1 = \{R,R\} Urn 2 = \{R,W\}
   b. Urn 1 = \{R,R\} Urn 2 = \{W,W\}
3. Compute \(P(\text{selecting two } R's)\) and \(P(\text{selecting at least one } R)\) in case (a) and case (b).

**Problem 3**
A recent poll on voting preferences in a local election involving two candidates was inconclusive, declaring that 50% of voters prefer candidate A and 50% prefer candidate B. However, the poll also found that in the group of voters who prefer A, \(a\)% are homeowners and \((100-a)\)% are not, while in the group of voters who prefer B, \(b\)% are homeowners, and \((100-b)\)% are not. We are interested in quantifying how homeownership status affects voting preference. Based on the results of the poll:
1. What is the probability that a randomly selected voter will be a homeowner, as a function of \(a\) and \(b\)?
2. Given that the randomly selected voter is indeed a homeowner, what is the probability that he will prefer A, as a function of \(a\) and \(b\)?
3. Under what conditions on \(a\) and \(b\) will this probability exceed 0.8?

**General Suggestion:** Think of probabilities computed as functions of parameters in a problem, and of how to work out equality or inequality conditions in terms of these parameters.

**Problem 4**
Consider two events \(E\) and \(F\) in a sample space \(S\). Assuming that both \(P(E)\) and \(P(F)\) are strictly > 0 and < 1, draw Van diagrams for situations in which \(E\) and \(F\) must provide information on one another. How does the situation change if one or both events are allowed to have probability 0, or if one or both of the events are allowed to coincide with the whole \(S\) (thus, have probability 1).

**Problem 5**
Consider \(n\) independent flips of a “magic” coin, with \(P(\text{head})=p\).
1. What is the probability of obtaining exactly one head?
2. What is the probability of obtaining at least one head?
3. What is the probability of obtaining exactly \(k\) heads? (generic \(k\) in \(\{1,2,...,n\}\))
4. What is the probability of obtaining at least \(k\) heads? (generic \(k\) in \(\{1,2,...,n\}\))
5. What is the probability of obtaining \(n\) heads?