Comment on “Model-based clustering for social networks” by Handcock et al

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The article by Handcock et al provides an interesting and important extension of the latent space model of Hoff et al (2002). A different extension of this work — which may also be applied to the current article — allows for more explicit modeling of local network features, such as transitivity, using an exponential random graph model (ERGM).

If the matrix \( y \) denotes the entire network (i.e., the collection of all \( y_{i,j} \)), then equation (2) of Handcock et al implies that

\[
P(Y = y) = \frac{\exp\{\beta_0^T \sum_{i,j} x_{i,j} y_{i,j}\} \exp\{\beta_1 \sum_{i,j} |z_i - z_j| y_{i,j}\}}{\kappa(\beta_0, \beta_1)},
\]

where \( \kappa(\beta_0, \beta_1) \) is a normalizing constant.

To simplify notation in (1), let

\[
\begin{align*}
g(y, X) &= \sum_{i,j} x_{i,j} y_{i,j} \\
h(y, Z) &= \sum_{i,j} |z_i - z_j| y_{i,j}.
\end{align*}
\]

Conditional on the latent positions \( Z \), the resulting model,

\[
P(Y = y) = \frac{\exp\{\beta_0^T g(y, X) + \beta_1 h(y, Z)\}}{\kappa(\beta_0, \beta_1)},
\]

is evidently a canonical exponential family (see, e.g., Lehmann, 1983) of distributions parameterised by \((\beta_0, \beta_1)\) with statistics \( g(y, X) \) and \( h(y, Z) \). Therefore, conditional on \( Z \), model (3) is an exponential random graph model (“graph” here is a synonym for “network”). Snijders (2002) and Robins et al (2006a) give literature reviews of these models, which are also called p-star models in the literature.
Importantly, model (3) is still an ERGM, conditional on $Z$, if the vector $g(y, X)$ of network statistics of interest is not of the form (2) that allows the likelihood function to factor nicely as in (1). The simplest such “non-factoring” models were considered by Frank and Strauss (1986), in which $g(y, X)$ contained terms such as the number of triangles in $y$, $\sum_{i<j<k} y_{i,j}y_{j,k}y_{k,i}$. Much recent work in the social networks literature has focused on development of useful statistics $g(y, X)$ for modeling real network data (Snijders et al., 2006; Robbins et al., 2006b), as well as explaining why some statistics, such as the number of triangles, lead to ERGMs that fail miserably at modeling these data (Handcock, 2002; Handcock, 2003).

Model (3) would give the modeler a powerful tool for exploring network structure: For instance, if the latent positions and cluster assignments of the nodes change dramatically upon the introduction of a particular network statistic into the ERGM, this suggests that the statistic captures an important aspect of network structure. Yet estimating parameters in a model such as (3) is quite difficult when $g(y, X)$ is not of the form (2). In principle, the two-stage maximum likelihood estimation of Handcock et al should work, though the second stage would rely on an MCMC-based stochastic algorithm such as those described by Hunter and Handcock (2006) or Snijders (2002). The Bayesian scheme implemented here is promising, but establishing a reasonable prior for the ERGM parameter $\beta_0$ is difficult. Despite the remaining challenges, the article of Handcock et al is a real step forwards.

References


