There are other methods

**Principal Factor Method** (and iterated versions of it):

\[ \hat{D}(\sigma_{j(1)}^2) \]

\[ S - \hat{D}(\sigma_{j(1)}^2) = \sum_{j=1}^{T} \lambda_{j,o(1)} V_{j,o(1)} V_{j,o(1)}' \]

\[ \hat{S}_{o(1)} = \sum_{j=1}^{K} \lambda_{j,o(1)} V_{j,o(1)} V_{j,o(1)}' \]

\[ \hat{\Lambda}_{(1)} = V_{o(1)} \Lambda_{o(1)}^{1/2} \]

\[ \hat{D}(\sigma_{j(2)}^2) = S - \hat{S}_{o(1)} \]

until the commonalities converge – if they do.

**Maximum likelihood method:**

Produce maximum likelihood estimates under the assumption that the data are Gaussian with a var/cov structure parameterized as

\[ \Delta \Delta' + D(\sigma_j^2) , \ \Delta \ T \times K \]

with a fixed \( K \). Sequential testing based on likelihood ratio to determine the appropriate \( K \).

**“Optimal” method(s):**

Similar to approach presented for the spherical case. Try to identify the “largest” diagonal matrix such that \( S - D(\sigma_j^2) \) is nnd. Proof of existence and uniqueness under regularity conditions, then numerical approximation. In this way \( K \) is naturally determined, and is minimal.
Rotations (for interpretation):

Given whichever $K$ and (non-unique) loading estimates, what is the right coordinate system in $K$ dimensions, i.e. the right rotation matrix to use in

$$X_i = \hat{\Lambda} \Theta F_i + e_i, \quad S = \hat{\Lambda} \Theta \Theta' \hat{\Lambda}' + D(\hat{\sigma}_f^2)$$

If possible, try to find a rotation that makes each point unambiguously closer to (only) one factor.

Then interpret factors as expressing groups of original coordinate variables.

Ex: Varimax rotation, and others.

If not possible, can try so called “oblique rotations”; actually general non-singular linear transformations identifying a basis that needs not be orthogonal, nor normal.

$T$ points (one for each original coordinate) in the $K$-dimensional space of the loadings.