Fisher’s exact test and Mantel-Haenszel test

The key to understanding Fisher’s exact test is that it’s identical to the randomization test back in chapter 4 except that the test statistic is the difference in proportions between two groups.

Thus, we consider ALL POSSIBLE reassignments of the individuals to the two groups and calculate \( \hat{\pi}_2 - \hat{\pi}_1 \) for each. The p-value of the test is the proportion of the reassignments giving more extreme values of \( \hat{\pi}_2 - \hat{\pi}_1 \) than observed.

A Mantel-Haenszel test involves the upper-left corner of a 2 \( \times \) 2 table. Calculate the expected value there in the usual way, then the test statistic is the excess, which is the value of observed minus expected. Under \( H_0 \), the excess is approximately normal with mean 0 and variance

\[
\frac{(\text{Row}_1)(\text{Row}_2)(\text{Column}_1)(\text{Column}_2)}{\text{(Total)}^2(\text{Total} - 1)}
\]

The nice thing about Mantel-Hanszel statistics is that they may be added (along with their variances) for several 2 \( \times \) 2 tables to give an overall \( Z \) statistic.