Question 1  Since the standard deviations are assumed known and the populations are normal, we use a z-test.

(a) The z-statistic is
\[ z = \frac{18.67 - 17.01}{\sqrt{\frac{1.62^2}{40} + \frac{1.42^2}{32}}} = 4.69. \]

The alternative is right-sided, so the p-value (the probability of seeing something more supportive of \( H_a \) than the observed 4.69) is the probability of a Z-value greater than 4.69. Since 4.69 is not in table A.3, we conclude that the p-value is smaller than the smallest area to the right listed in table A.3, which is .0002. (Using a computer, the p-value may be found to be .00000137.)

(b) Using the first formula at the bottom of p. 365, we plug in \( \Delta_0 = 0 \), \( \Delta' = 1 \), \( z_\alpha = 1.645 \), and \( \sigma = \sqrt{\frac{1.62^2}{40} + \frac{1.42^2}{32}} = .354 \). We obtain
\[ \beta = \Phi \left( 1.645 - \frac{1-0}{.354} \right) = \Phi(-1.18) = .1190. \]

Thus, the power, which is \( 1 - \beta \), is .8810.

Alternatively, we can reason as follows: The power is the probability that we reject \( H_0 \) if the truth is \( \Delta_0 = 1 \). But we reject whenever \( Z > 1.645 \). Writing \( Z = \frac{\overline{X} - \overline{Y}}{\sigma} \), we reject whenever \( \overline{X} - \overline{Y} > 1.645\sigma \). Thus, under the assumption that \( \overline{X} - \overline{Y} \) is normal with mean 1 and standard deviation \( \sigma \), we get
\[ \text{power} = P \left( \frac{\overline{X} - \overline{Y} - 1}{\sigma} > \frac{1.645\sigma - 1}{\sigma} \right) = P \left( Z > 1.645 - 1 \right), \]
which is the same as above.

(c) If the true standard deviations were unknown, we’d use a t-test instead of a z-test above. The t-statistic would be calculated in the same way as the z-statistic, so we would obtain \( t = 4.69 \). The degrees of freedom would be calculated using the formula following equation (9.2) on p. 373, and the p-value would be very small just as it was above. (One could also use a pooled t-test here, which would change the t-statistic a bit, but the end result would still be a very small p-value.)

Question 2  (a) Using a z-test because both samples are large, we get
\[ Z = \frac{123.2 - 109.7 - 10}{\sqrt{\frac{1.3^2}{140} + \frac{2.0^2}{140}}} = 17.36. \]

With a right-sided alternative, this positive Z-statistic is incredibly extreme, way outside Table A.3, so the p-value is smaller than the smallest area to the right listed in the table, which is .0002. (In fact, a computer gives the p-value as \( 8.2 \times 10^{-68} \).)

(b) The 99% confidence interval will be based on \( z_{.005} = 2.575 \). The interval equals
\[ 123.2 - 109.7 \pm 2.575 \sqrt{\frac{1.3^2}{140} + \frac{2.0^2}{140}} = 13.5 \pm .52, \]
which means the interval stretches from 12.98 to 14.02.

Question 3  (a) The pooled variance is
\[ s_p^2 = \frac{5}{12} \times 11.3^2 + \frac{7}{12} \times 8.3^2 = 93.39. \]

(b) With and without the pooled variance, respectively, the standard error is
\[ \sqrt{\frac{93.39}{6} + \frac{93.39}{8}} = 5.22 \quad \text{and} \quad \sqrt{\frac{11.3^2}{6} + \frac{8.3^2}{8}} = 5.47. \]
(c) \( t = (40.3 - 21.4)/5.47 = 3.46. \)

(d) Estimated degrees of freedom are given by

\[
\frac{(5.47^2)}{\left(\frac{11.3^2}{6} + \frac{8.3^2}{5}\right)} = \frac{895.3}{452.9 + 14.7} = \frac{895.3}{101.2} = 8.85.
\]

Round this down to 8 degrees of freedom.

(e) The area to the right of 3.46 for a t-distribution on 8 degrees of freedom is (from Table A.8) .004. Since the alternative hypothesis is two-sided, we double this to obtain the p-value of .008.

**Question 4** This problem is a paired t-test problem because there is a natural pairing of the \( X \)'s with the \( Y \)'s (or in this case, the unabradeds with the abradeds). The sample of differences (\( U \) minus \( A \)) is

\[7.9 \quad 35.0 \quad 5.5 \quad 4.2 \quad 6.7 \quad -3.7 \quad -0.9 \quad 3.3\]

This sample has mean 7.25 and standard deviation 11.86. Thus, the t-statistic equals 7.25/(11.86/\( \sqrt{8} \)) = 7.3. For a right-sided alternative, we need the area to the right of 1.73. For 7 degrees of freedom, table A.8 gives this area as .065.

**Question 5** (a) We have \( \hat{p}_1 = \frac{63}{300} = .210 \) and \( \hat{p}_2 = \frac{75}{180} = .417 \). Also, \( m = 300 \), \( n = 180 \), and the pooled estimate of \( p \) is \( \hat{p} = \frac{300 + 180}{300 + 180} = .2875 \). Thus,

\[
Z = \frac{.21 - .417}{\sqrt{.2875(1 - .2875) \left( \frac{1}{300} + \frac{1}{180} \right)}} = -4.85.
\]

The p-value for the 2-sided alternative is twice the probability to the left of -4.85, which is too small for Table A.3. Thus, we conclude that the p-value is smaller than twice the smallest value in the table, or smaller than .0004.

(b) If we take \( p_1 = .2 \) and \( p_2 = .4 \), then we obtain \( \bar{p} = (300 \times .2 + 180 \times .4)/480 = .275 \). Furthermore,

\[
\sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{300} + \frac{\hat{p}_2(1 - \hat{p}_2)}{180}} = .0432 \quad \text{and} \quad \sqrt{.275(1 - .275) \left( \frac{1}{300} + \frac{1}{180} \right)} = .0421.
\]

Thus, from the formula on p. 394, we obtain

\[
\beta = \Phi \left[ \frac{1.96 \times .0421 - (.2 - .4)}{.0432} \right] - \Phi \left[ \frac{-1.96 \times .0421 - (.2 - .4)}{.0432} \right] = \Phi(6.54) - \Phi(2.72).
\]

Since \( \Phi(6.54) \) is approximately 1 (it’s way off the table), we get \( \beta = 1 - \Phi(2.72) = .0033 \). Since \( \beta \) is the probability that \( H_0 \) will NOT be rejected under the assumed alternative, the probability that it WILL be rejected (also known as the power) equals .9967.

**Question 6** Part (a) was not assigned.

(b) Since the parameter of interest is \( p_3 - p_2 \), the estimator is \( (X_3 - X_2)/n \). Setting \( \hat{p}_3 = X_3/n \) and \( \hat{p}_2 = X_2/n \), this can be expressed as \( \hat{p}_3 - \hat{p}_2 \).

(c) The test statistic will be

\[
Z = \frac{\hat{p}_3 - \hat{p}_2}{\sqrt{\frac{\hat{p}_3 + \hat{p}_2 - (\hat{p}_3 - \hat{p}_2)^2}{n}}}.
\]

(d) We have \( \hat{p}_3 = \frac{200}{1000} = .20 \) and \( \hat{p}_2 = \frac{150}{1000} = .15 \). Thus,

\[
Z = \frac{.2 - .15}{\sqrt{\frac{.2 + .15 - (2 - 1.5)^2}{1000}}} = \frac{.05}{\sqrt{.0003475}} = 2.68.
\]

For the 1-sided alternative of this problem, this z-statistic gives a p-value of .0037, which is strong evidence against the null hypothesis.