Question 1 (a)

\[ H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 \]
\[ H_a : \text{Not all of } \mu_1, \mu_2, \mu_3, \text{ and } \mu_4 \text{ are the same.} \]

(b) The null hypothesis is that all four types of iron formation have the same total iron on average. The alternative hypothesis is that the four averages are not all the same.

(c) The ANOVA table is given below as it was produced by Minitab:

```
Analysis of Variance for Fe
Source    DF   SS     MS      F      P
formatio  3  509.1  169.7  10.85  0.000
Error     36  563.1  15.6
Total     39 1072.3
```

(d) With 3 and 36 degrees of freedom, the .01 critical F value is approximately 2.9. Since the observed F statistic of 10.85 is much larger than this critical value, we reject \( H_0 \) at the .01 level (alternatively, we note that the p-value reported by Minitab is smaller than .01, which gives the same conclusion).

We conclude that the four types of iron formation do not all have the same mean amount of iron.

Question 2 (a) The treatment factor had 5 levels because there are 4 degrees of freedom for the treatment effect (the number of treatment df is always one less than the number of treatment levels).

(b) As seen from the filled-in table below, the estimate of the common variance, also known as the mean square error, equals 1073.8.

(c)

```
Analysis of Variance for Response
Source    DF   SS     MS      F
Treatment  4  44318  (3) (5)
Error     (1) (2) (4)
Total     34  76533
```

(1) Since the treatment DF plus the Error DF equals the total DF, we conclude that the error DF equals 30 in this case.

(2) Since the treatment SS plus the Error SS equals the total SS, we conclude that the error SS equals 76533 - 44318 = 32215.

(3) Since the MS column equals SS divided by DF, the treatment MS equals 44318/4 = 11079.5.

(4) Since the MS column equals SS divided by DF, the error MS equals 32215/30 = 1073.8.

(5) Since the F statistic is the ratio of the MS values, the F statistic equals 11079.5/1073.8 = 10.32.
**Question 3** The Tukey confidence intervals are given below:

Intervals for (column level mean) - (row level mean)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-3.375</td>
<td>6.155</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-8.635</td>
<td>-10.025</td>
<td>0.895</td>
</tr>
<tr>
<td>4</td>
<td>-12.525</td>
<td>-13.915</td>
<td>-8.655</td>
</tr>
</tbody>
</table>

Three of the confidence intervals above do not contain zero, indicating significant differences at the .05 level. Since all three of these intervals are negative, the column level mean is significantly smaller than the row level mean in each case. We conclude that $\mu_2 < \mu_3$, $\mu_2 < \mu_4$, and $\mu_1 < \mu_4$.

**Question 4 (a)** The overall mean is

$$\frac{7}{23} \times 8.414 + \frac{5}{23} \times 7.480 + \frac{5}{23} \times 6.380 + \frac{6}{23} \times 5.767 = 7.078.$$  

(b) The pooled variance is

$$\frac{6}{19} \times 1.145^2 + \frac{4}{19} \times 1.702^2 + \frac{4}{19} \times 1.065^2 + \frac{5}{19} \times 1.558^2 = 2.120.$$  

(c) In order to carry out ANOVA, we must assume that folicin content is normally distributed in each of the four brands and that the variance of folicin content is the same in each of the four brands.

(d) For the DF column, four treatment levels means 3 treatment degrees of freedom; 23 total observations means 22 total degrees of freedom; and the difference of 19 means 19 error degrees of freedom.

We already found the error MS in part (b), which is 2.120.

Finally, using the overall mean from part (a), we get


We now have all we need to fill in the table. The MSTr is the SStr divided by the DFT, and the SSE is the MSE times the DFE. Finally, the F statistic is the MSTr divided by the MSE. The resulting ANOVA table looks like this:

**Analysis of Variance for Response**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>3</td>
<td>26.051</td>
<td>8.683</td>
<td>4.10</td>
</tr>
<tr>
<td>Error</td>
<td>19</td>
<td>40.280</td>
<td>2.120</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>22</td>
<td>66.331</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(If you’re curious, this F statistic means that the p-value lies between .01 and .05 according to Table A.9.)