1. The probability that an American engineering firm will establish a branch office in Toronto is .7, the probability that it will establish a branch office in Mexico City is .4 and the probability it will establish a branch office in at least one of the cities is .8. Define events A and B as follows:
   \[ A = \{ \text{American engineering firm will establish a branch office in Toronto} \} \]
   \[ B = \{ \text{American engineering firm will establish a branch office in Mexico City} \} \]

   It may be useful to draw Venn diagrams. For parts (a)-(c), what is the probability that a branch office will be established in
   
   (a) in both cities?
   (b) neither of the cities?
   (c) exactly one of the cities?
   (d) Are events A and B independent? Why or why not?

2. From a load of 50 Panasonic Tape recorders 35 are destined for New York and 15 for Boston. If two are shipped to Trenton by mistake and the “selection” is random, what are the probabilities that
   
   a) both tape recorders should have gone to New York;
   b) one should have gone to New York and one to Boston.

3. From 5 biologists, 4 chemists, and 3 physicists,
   
   (a) how many committees of size 4 can be formed?
   (b) how many committees containing 2 biologists, 1 chemist, and 1 physicist can be formed?

4. Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Define events A and B as follows:
   \[ A = \{ \text{light aircraft that disappears is discovered} \} \]
   \[ B = \{ \text{light aircraft that disappears has an emergency locator} \} \]

   Suppose a light aircraft has disappeared.
   
   (a) What is the probability that it has an emergency locator and it will not be discovered?
   (b) What is the probability that it has an emergency locator?
   (c) If it has an emergency locator, what is the probability that it will not be discovered?
5. Grafting, the uniting of the stem of one plant with the stem or root of another, is widely used commercially to grow the stem of one variety that produces fine fruit on the root system of another variety with a hardy root system. For example, most sweet oranges grow on trees grafted to the root of a sour orange variety. Suppose that each graft fails independently with probability .3.

a) Write the formula for the p.m.f. for $X$, the number of grafts that fail in a series of five trials.

b) Plot the cumulative distribution function $F$.

c) Suppose that the cost of each failed graft is $9.00. Find

(i) the probability that the cost from failed grafts will exceed $20.00 in the five trials.
(ii) the expected cost from failed grafts in the five trials.
(iii) the variance of the cost from the failed grafts in the five trials.

6. Let $X$ denote the amount of time for which the Stat 401 book on two-hour reserve at McAllister is checked out by a randomly selected student, and suppose that $X$ has density function

$$f(x) = \begin{cases} 0.5x & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that the book is checked out between .5 and 1.5 hours?

7. The finished inside diameter of a piston ring is normally distributed with a mean of 10 cm and a standard deviation of 0.03 cm.

(a) Above what value of inside diameter will 85.08% of the piston rings fall?
(b) What is the probability that the diameter of a randomly selected piston will be less than 10.06?

8. Suppose your waiting time for the bus in the morning has mean 3 minutes and variance $28\frac{25}{25}$ minutes, while the waiting time in the evening has mean 6 minutes and variance 4. In a typical week, you take the bus 5 times in the morning and 3 times in the evening.

(a) Calculate the expected value of the total waiting time in a typical week.
(b) Calculate the variance of the total waiting time in a typical week. State your assumptions.

**Hint:** Set $X_i =$ ith morning waiting time where $i=1,...,5$ and $Y_j =$ jth evening waiting time where $j=1,2,3$.

9. A question relating to a study of the echo-location system for bats is how far apart are the bat and an insect when the bat first senses the insect. The technical problems for measuring this are complex and so only 11 data points, $Y_1,\ldots,Y_{11}$, were obtained. It is given that $\bar{y} = 48.36$ and $s^2 = 327.05$.

(a) Construct a 95% confidence interval for the mean distance $\mu$.
(b) What assumption did you use?
10. An appliance manufacturer is considering the purchase of a new machine for cutting sheet metal parts. If $\mu_0$ is the average number of metal parts cut per hour by her old machine and $\mu$ is the corresponding average for the new machine, the manufacturer wants to test the null hypothesis $H_0 : \mu = \mu_0$ against a suitable alternative. What should the alternative be if
   (a) she does not want to buy the new machine unless there is evidence it is more productive than the old one;
   (b) she wants to buy the new machine (which has some other nice features) unless there is evidence it is less productive than the old one?
   (c) For each of the two cases above state whether she should buy the new machine if the null hypothesis gets rejected.
   (d) Suppose that $\mu_0 = 9.5$ cuts per hour and she decides to test $H_0 : \mu = 9.5$ against $H_a : \mu > 9.5$ at the $\alpha = .05$ level. She gets the machine manufacturer to lend her a new machine and she measures the number of cuts made by the machine in fifty (50) randomly selected one hour time periods. The summary statistics for this random sample are $\bar{x} = 9.8$ and $s = 1.095$. Would you reject $H_0$?

11. A study of the electromechanical protection devices used in electrical power systems showed that of 193 devices that failed when tested 75 where due to mechanical parts failures.
   (a) Find a 95% confidence interval for $p$, the proportion of failures due to mechanical causes.
   (b) How large a sample is required to estimate $p$ within 0.03, i.e. to have a confidence interval of length 0.06, with 95% confidence;
      (i) when you have no idea of what $p$ might be;
      (ii) when a preliminary study investigations leads you to believe that $p \approx .39$

12. A manufacturer of automatic washers provides a particular model in one of three colors, white, almond, or blue. Of the first 1000 washers sold, it is noted that 400 of the washers were white. The manufacturer wants to know if this is evidence that more than a third of consumers prefer white washers.
   (a) Set up the appropriate null and alternative hypotheses for the manufacturer.
   (b) Carry out a test of your null hypothesis versus your alternative hypothesis at the $\alpha = .05$ level. Is there evidence that more than one third of consumers prefer white washers?
   (c) What is the approximate $P$-value of your test?

13. A decision is to be made whether to site an incinerator for the county in Placerville or Centreville. The county commissioners favor Centreville. However, the residents of Centreville claim that a much larger percentage of residents in Centreville are strongly opposed to the incinerator being situated in their town than the corresponding percentage of residents of Placerville. (That is, the percentage of residents of Placerville that are strongly opposed to the incinerator being situated in Placerville.) The county commissioners agree to place the incinerator in Placerville, if a survey shows almost conclusive evidence in support of the Centreville residents’ contention.
Suppose 330 of the 890 residents of Centreville strongly oppose the incinerator being situated in Centreville. Suppose 170 of the 550 residents of Placerville strongly oppose the incinerator being situated in Placerville. Does this survey provide the evidence that the Centreville residents hoped for?

14. Publication of the national average score on the Scholastic Aptitude Test (SAT’s) each year is often the occasion for commentators to claim that American high school students are learning more (if the scores are up) or less ( if the scores are down). Below are the average SAT scores and the sample standard deviations for five randomly chosen school districts over the twelve years 1976 to 1987. \( \bar{x} = 427.75 \) and \( \bar{s} = 2.05 \).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>431</td>
<td>429</td>
<td>429</td>
<td>427</td>
<td>424</td>
<td>426</td>
<td>425</td>
<td>426</td>
<td>431</td>
<td>431</td>
<td>431</td>
<td>430</td>
</tr>
<tr>
<td>( s )</td>
<td>2.11</td>
<td>2.09</td>
<td>1.98</td>
<td>2.13</td>
<td>2.02</td>
<td>1.97</td>
<td>2.10</td>
<td>2.11</td>
<td>2.13</td>
<td>1.96</td>
<td>2.04</td>
<td></td>
</tr>
</tbody>
</table>

(a) Make a control chart for these scores.

(b) Are any of the points out of control? Which ones?

(c) Based on the Western Electric supplemental rules is there any evidence that the process is out of control?

15. Twenty-two runs of a batch chemical process were conducted. Twelve of the runs used one catalyst (catalyst 1), the remaining ten runs used a second catalyst (catalyst 2). The twelve runs with catalyst 1 provided an average yield of 85 grams, with a sample standard deviation of 4 grams. The ten runs with catalyst 2 provided an average yield of 81 grams, with a sample standard deviation of 5 grams.

(a) It has been conjectured that catalyst 1 is the better of the two. State the null hypothesis and alternative hypothesis you would use for testing this conjecture.

(b) Perform the test, with level 5%, for the hypotheses described in part (a), and, in plain words, state your conclusion about the effectiveness of catalysts 1 and 2.

16. One of six different machines is to be used for manufacturing rubber seals. Because the cost and complexity of operating the machines differ, it is not enough to simply choose the machine that produces the strongest rubber seals. Instead, as an initial step towards making a decision about which machine to use, an experiment is conducted to determine which machines produce rubber with different tensile strengths. A random sample of 4 seals is collected from each of the 6 machines. The following data are in units of kilograms per square centimeter \( \times 10^{-1} \).

<table>
<thead>
<tr>
<th>Machines</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.0</td>
<td>16.4</td>
<td>20.8</td>
<td>14.6</td>
<td>18.0</td>
<td>15.3</td>
<td></td>
</tr>
<tr>
<td>15.4</td>
<td>19.2</td>
<td>16.2</td>
<td>16.7</td>
<td>19.7</td>
<td>13.2</td>
<td></td>
</tr>
<tr>
<td>14.3</td>
<td>17.7</td>
<td>18.3</td>
<td>20.8</td>
<td>17.0</td>
<td>14.5</td>
<td></td>
</tr>
<tr>
<td>17.1</td>
<td>15.4</td>
<td>19.4</td>
<td>18.9</td>
<td>21.0</td>
<td>17.1</td>
<td></td>
</tr>
<tr>
<td>15.70</td>
<td>17.18</td>
<td>18.67</td>
<td>17.75</td>
<td>18.93</td>
<td>15.03</td>
<td></td>
</tr>
</tbody>
</table>

\begin{align*}
\text{mean} & = 15.03 \\
\text{sum of squares} & = 910.99 \\
\text{Variances} & = 1.37 \\
\end{align*}
(a) State the null hypothesis and alternative hypothesis you would use for testing whether the machines A-F produce rubber seals with the same tensile strength.
(b) Perform a test of the hypotheses stated in (a), with level 5%.
(c) Explain what is meant by *multiple comparisons* and *experimentwise error rate*
(d) For which pairs of machines do you conclude that the mean tensile strength is different?

17. Devore problem 10.16

18. Answer Devore problem 11.20 using the ANOVA table below:

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>1</td>
<td>13338.9</td>
<td>13338.9</td>
<td>192.08</td>
<td>0.000</td>
</tr>
<tr>
<td>Phosphor</td>
<td>2</td>
<td>1244.4</td>
<td>622.2</td>
<td>8.96</td>
<td>0.004</td>
</tr>
<tr>
<td>Interaction</td>
<td>2</td>
<td>44.4</td>
<td>22.2</td>
<td>0.32</td>
<td>0.732</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>833.3</td>
<td>69.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>17</td>
<td>15461.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

19. Devore problem 12.32