Feb. 25 (False) Statistic for the day: Every 1 second in the U.S., a student in public school is suspended.

- from a child abnormal psych textbook

Slightly more reputable: Total number of suspensions of public elementary and secondary students in US in 2000: 3,053,449

Assignment: Read Chapter 17
Exercises pp. 330-333: 2, 4, 6, 13, 18

Rule 4: If the occurrence of one event forces the occurrence of another event, then the probability of the second event is always at least as large as the probability of the first event.

If event A forces B to occur, then \( P(A) \leq P(B) \)

Special case: \( P(E \text{ and } F) \leq P(E) \)
\[ P(E \text{ and } F) \leq P(F) \]

Aids Example 12, p. 306

Suppose the probability of getting infected by HIV from a single heterosexual encounter without condom is 0.002.

Then the probability of not infected is \( 1 - 0.002 = 0.998 \) \text{ Rule 1}

Probability of at least one infection in 10 independent encounters:
\[ P(\text{at least one infection in 10}) = 1 - P(\text{no infections in 10}) \] \text{ New Rule}

But \( P(\text{no infections in 10}) = 0.998^{10} \) raised to the 10th power.
\[ = 0.9802 \] \text{ Rule 3}

So \( P(\text{at least one infection in 10}) = 1 - 0.9802 = 0.0198 \) \text{ New Rule}

Mary likes earrings and spends time at festivals shopping for jewelry. Her boy friend and several of her close girl friends have tattoos. They have encouraged her to also get a tattoo.

Unknown to you, Mary will be sitting next to you in the next stat 100.2 class.

Rank the following statements from most likely to least likely:
A. Mary is a physics major.
B. Mary is a physics major with pierced ears.
C. Mary has pierced ears.

New Rule:

Suppose we are considering a series of events. The probability of at least one of the events occurring is:
\[ P(\text{at least one}) = 1 - P(\text{none}) \]

This follows directly from Rule 1 since ‘at least one’ or ‘none’ has to occur.

Digression: Birthday problem

- Given 40 people, what is the probability of at least one matching set of birthdays?

- Easier question: What is the probability of no matching sets of birthdays?

- Answer: \( 0.1087 \) (not necessary to understand how this is found unless you’re just curious)

- Thus, \( P(\text{at least one matching set of birthdays}) = 1 - 0.1087 \) which is \( 0.8913 \)

With 180 people, the probability of no matching birthdays is \( 0.00000000000000000373 \)
Long Run Behavior

We CANNOT predict individual outcomes.

BUT

We CAN predict quite accurately long run behavior.

Standard example:

We cannot predict the outcome of a single toss of a coin very precisely: \( P(\text{head}) = .50 \)

But in the long run we expect about 50% heads and tails.

Two laws (only one of them valid):

- Law of large numbers: Over the long haul, we expect about 50% heads (this is true).
- “Law of small numbers”: If we’ve seen a lot of tails in a row, we’re more likely to see heads on the next flip (this is completely bogus).

Remember: The law of large numbers OVERWELMS; it does not COMPENSATE.

Margin of error for a coin toss

Recall from Chapter 4: If we have a simple random sample of a poll with answers yes or no. Then the proportion of yeses will have a margin of error equal to \( \frac{1}{\sqrt{n}} \).

Our poll consists of tossing a coin, say 10 times, and recording the number of heads (yeses). We expect the proportion of heads to be around .5.

What do we mean by around .5? We mean within a margin of error: \( \frac{1}{\sqrt{10}} = .32 \)

So we could see if our proportion of heads is within .5 ± .32 or between .18 and .82.
Suppose you contact them one at a time and independently.

From the 2001 survey: 60% of the women said no they don’t have someone who does not have a steady boy friend.

How many women will you have to contact before you find a steady boy friend.

Consider the “odd man” game. Three people at lunch toss a coin. The odd man has to pay the bill.

When will it happen? Odd Man

You are the odd man if you get a head and the other two have tails or if you get a tail and the other two have heads.

P(no odd man) = P(HHH or TTT)

= P(HHH) + P(TTT) Rule 2

= (1/2)^3 + (1/2)^3 Rule 3

= 1/8 + 1/8

= 1/4 = .25

P(odd man) = 1 – P(no odd man) = 1 - .25 = .75 Rule 1

When? How soon?

P(1st try) = .60

P(2nd try) = P(not available then yes available)

= .40x.60 = .24 assume independence and Rule 3

P(3rd try) = P(not and not and then yes available)

= .40x.40x.60 = .096

P(on or before the 3rd try) = .60 + .24 + .096 = .936

P(have to ask more than 3) = 1 - .936 = .064

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**Expectation**

**Insurance**

Example 14, p. 307 extended.

Suppose my insurance company has 10,000 policy holders and they are all skateboarders.

I collect a $500 premium each year.

I pay off $1500 for a claim of a skate board accident.

From past experience I know 10% will file a claim.

How much do I expect to make per customer?

P(claim) = .10  loss is $1500 - $500 = $1000 recorded as -$1000

P(no claim) = .90  gain is $500

Expected value = .10(-1000) + .90(500) = -100 + 450 = 350 dollars per customer

Expected value for the 10,000 customers

= 10,000x350

= 3,500,000 dollars per year

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Alternatively:

Expect 10% or 1,000 claims for 1000x(-$1000) = -$1,000,000 loss

Expect 90% or 9,000 earning $500 each or $4,500,000 gain

So we expect $4,500,000 - $1,000,000 = $3,500,000 net profit

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Some old ideas from chapter 7 (histograms) and chapter 8 (bell shaped curves).

Histogram of net profits over many years might look like this.

The standard deviation is roughly

\[(3,600,000 - 3,400,000)/4 = 50,000\]

Actual value is 45,000 computed from a formula.

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So 95% of the time the net profit will be between $3,410,000 and $3,590,000 with expected value $3,500,000

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Football slips
Consider Tenn vs So. Carolina +21
If you pick Tennessee then they must win by more than 21 points for you to win.
If you pick So. Carolina then if they are beaten by less than 21 points then you win.
So if the score is Tenn 48  So. Carolina 21 you win if you picked Tenn.

Suppose you play the 6 out of 6 game.
You pay $1.
If you pick 6 out of 6 correctly then you get $28.
From the point of view of the professional gambler who sold you the slip:

P(you win) = (1/2)^6 = .015625 Rule 3
P(pro gambler wins) = 1 - .015625 = .984375 Rule 1

Pro gambler expectation per customer:

\[-27x.015625 + 1x.984375 = .5625\]
or 56.25 cents on every dollar

Gambler keeps your $1

Suppose the pro gamblers sell 10,000 6 out of 6 slips
The expected gain is 10000x.5625 = $5625

Hence 95% of the time the pro gamblers selling 10,000 6 out of 6 slips at $1 each expect to gain:

$5625 + $700
$4925 to $6325

Suppose the pro gamblers sell 10,000 6 out of 6 slips
The expected gain is 10000x.5625 = $5625

Standard deviation roughly equal to (6300-4900)/4 = $350