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STAT 401, Section 1

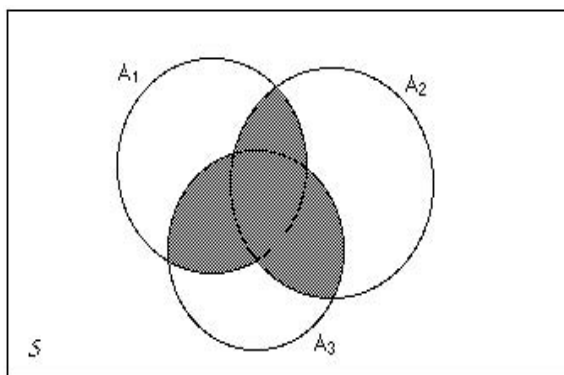
Mid-term Examination #1

Spring, 2004

**Print Name:** \_\_\_\_\_ **Sign Name:** \_\_\_\_\_

- (1) PRINT (in block letters) YOUR FULL NAME AND SIGN on the lines above. You must hand in your exam packet; you will be able to pick it up later. (5 points off for students who do not sign and turn in an exam packet.)
- (2) PRINT (in block letters) YOUR NAME on your scantron sheet (5 points off for students who do not follow these instructions.)
- (3) PRINT AND ENTER YOUR STUDENT NUMBER on your scantron sheet (5 points off for students who enter an incorrect number.)
- (4) ENTER THE FORM of your exam packet, shown at the top of this page, on your scantron sheet. (5 points off for students who enter an incorrect form.)
- (5) WORK QUICKLY. There are 40 questions and you have 50 minutes.
- (7) Each exam question has only one correct answer. Identify the answer by filling in the appropriate slot on your scantron sheet.

### Question 1.



Suppose that three coins are flipped and that  $A_i$  denotes the event that coin  $i$  comes up heads ( $i = 1, 2, 3$ ). Which of the following events is represented by the Venn diagram above?

- (A) One or more of the coins come up heads
- (B) At least two of the coins come up heads
- (C) Exactly two of the coins come up heads
- (D) All three of the coins come up heads

**Question 2.** In the previous question, suppose that the coins 1 and 3 are fair coins but that coin 2 is a weighted coin that comes up heads with probability  $\frac{1}{3}$ . What is the probability of the event depicted in the Venn diagram?

- (A)  $\frac{1}{2}$
- (B)  $\frac{4}{7}$
- (C)  $\frac{7}{12}$
- (D)  $\frac{5}{12}$

**Question 3.** Suppose that a fair 6-sided die is rolled. Let  $X$  equal the number shown on the die (that is,  $P(X = i) = \frac{1}{6}$  for  $i = 1, \dots, 6$ ). What is  $V(X)$ ?

- (A)  $\frac{91}{6} - 3.5 = 11.67$   
 (B)  $\frac{91}{6} - \frac{91}{6} = 0$   
 (C)  $\frac{91}{6} - 3.5^2 = 2.917$   
 (D)  $\frac{91}{6} = 15.17$

**Question 4.** Suppose that a trick 6-sided die is weighted so that the probability  $x$  comes up equals  $k(1 + x)$  for  $x = 1, \dots, 6$ . What is the value of  $k$ ?

- (A)  $\frac{6}{21}$   
 (B)  $21$   
 (C)  $\frac{1}{21}$   
 (D)  $\frac{1}{27}$

**Question 5.** Suppose that  $X$  and  $Y$  are both Bernoulli random variables, with  $E(X) = \frac{1}{2}$  and  $E(Y) = \frac{1}{5}$ . The joint pmf of  $X$  and  $Y$  is given by

		$y$	
		$0$	$1$
$x$	$0$	$\frac{4}{10}$	$\frac{1}{10}$
	$1$	$\frac{4}{10}$	$\frac{1}{10}$

What is the absolute value of  $\text{Cov}(X, Y)$ ?

- (A)  $1$   
 (B)  $\frac{1}{10}$   
 (C)  $0$   
 (D)  $10$

**Question 6.** Suppose that 23% of all people believe that UFO's exist. If 5 people are selected at random, what is the probability that 2 of them believe in UFO's?

- (A)  $\frac{\binom{5}{2}\binom{23}{3}}{\binom{23}{5}} = .526$   
 (B)  $\binom{5}{2} (.23)^2 (.67)^3 = .242$   
 (C)  $\frac{\binom{5}{2}\binom{23}{3}}{\binom{5}{5}} = .526$   
 (D)  $\left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^3 = .035$

**Question 7.** Suppose that  $Y$  is a normal random variable with mean 5 and variance 4. What is  $P(3 < Y < 7)$ , rounded to the nearest tenth of a percent?

- (A) 50.0%  
 (B) 99.7%  
 (C) 95.4%  
 (D) 68.3%

**Question 8.** Suppose that  $X_1, \dots, X_{10}$  are independent normal random variables, each with mean 1 and variance 10. Let  $\bar{X}$  denote the sample mean  $\frac{1}{10} \sum_{i=1}^{10} X_i$ . If  $\mu_{\bar{X}}$  and  $\sigma_{\bar{X}}^2$  denote the mean and variance of  $\bar{X}$ , what is  $\mu_{\bar{X}} + \sigma_{\bar{X}}^2$ ?

- (A) 10
- (B) 11
- (C) 101
- (D) 2

**Question 9.** Suppose that  $X$  is a random variable with probability density function given by

$$f(x) = \begin{cases} ke^{-2x} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Let  $\mu = E(X)$ . What is the value of  $\mu + k$ ?

- (A) 1
- (B) 4
- (C) 2.5
- (D) 0.5

**Question 10.** Suppose that  $X$  is an exponential random variable whose mean equals 2. What is  $P(X > 4 | X > 2)$ ?

- (A)  $e^{-4}$
- (B)  $e^{-1}$
- (C)  $e^{-3}$
- (D)  $e^{-2}$

**Question 11.** Assume that you are given the following information about events  $A$ ,  $B$ , and  $C$ :

$$\begin{aligned} P(A) &= .55 & P(B) &= .65 & P(C) &= .7 \\ P(A \cup B) &= .9 & P(A \cup C) &= .9 & P(B \cup C) &= .85 \\ P(A \cap B \cap C) &= .2 \end{aligned}$$

What is the probability that exactly two of the three events  $A$ ,  $B$ , and  $C$  occur?

- (A) .60
- (B) .70
- (C) .55
- (D) .65