STAT 220: Basic Statistics for Quantitative Students

Spring 2006

Assignment due Mar. 24

7.18 **a.** BY, BS, BA, YS, YA, SA.
b. 1/6.

7.36 **a.** P(A) = .55; P(A^c) = .45; P(B|A) = .80; P(B|A^c) = .10.
b. P(A and B) = P(A)P(B|A) = (.55)(.80) = .44. This is the probability of being a Republican and voting for Candidate X.
c. P(A^c and B) = P(A^c) P(B|A^c) = (.45)(.10) = .045. This is the probability of being a non-Republican and voting for Candidate X.
d. P(B) = P(A and B) + P(A^c and B) = .485.
e. Candidate X received 48.5% of the votes.

7.49 There are 49 possible pairs, of which 7 have the same number twice, so the probability is 7/49 = 1/7.

8.60 **a.** μ = 100(1/2) = 50 and $\sigma = \sqrt{100(.5)(1-.5)} = 5$.
b. P(X ≥ 60) ≈ P(Z ≥ (60 − 50)/5) = P(Z ≥ 2) = 1 – P(Z < 2) = 1 − .9772 = .0228.
c. P(X ≥ 59.5) ≈ P(Z ≥ (59.5 – 50)/5) = 1 – P(Z < 1.9) = 1 − .9713 = .0287.

8.63 **a.** *X* is a binomial random variable. There are a fixed number of trials since *n* = 100. There are two possible outcomes—picking the correct suit and picking an incorrect suit. The outcomes are independent from one trial to the next because the cards are selected with replacement. The probability of picking the correct suit remains the same from trial to trial and *p* = 1/4 if the subject is just guessing.
b. If the subject is just guessing, the mean is $\mu = np = 100 \times .25 = 25$ and the standard deviation is $\sigma = \sqrt{np(1-p)} = \sqrt{100 * .25 * .75} = 4.33$.
c. For 33 correct guesses, $z = \frac{33-25}{4.33} = 1.85$ and $P(Z \geq 1.85) = 1 - P(Z \leq 1.85) = 1 - .9678 = .0322$. *Note*: Use of the binomial distribution with *n*=100 and *p*=.25 gives the exact answer, which is $P(X \geq 33) = 1 - P(X \leq 32) = 1 - .9554 = .0446$.
d. For 50 correct guesses, $z = \frac{50-25}{4.33} = 5.77$.
So, $P(X \geq 50) \cong P(Z \geq 5.77) = 1 - P(Z \leq 5.77) \cong 1 - .99999999 = .00000001$. The answer is about 1 in 100 million. It is nearly impossible to guess this well so getting 50 correct could be interpreted as evidence that a person is doing something other than randomly guessing.

B. Full house: There are 2 distinct values in a full house, 1 of which is the "triple". Once we take the suits of the triple and the pair into account, we have

$$C(13,2)C(2,1)C(4,2)C(4,3) = (78)(2)(6)(4) = 3744 \text{ hands.}$$

Three-of-a-kind: There are 3 distinct values, one of which is the "triple". Choosing values and suits gives

$$C(13,3)C(3,1)C(4,1)C(4,1)C(4,3) = (286)(3)(4)(4)(4) = 54912 \text{ hands.}$$

Flush: After choosing the suit, we just need to choose 5 values from the 13. However, we need to eliminate the 10 possible straights for each suit:

$$C(4,1)[C(13,5)-10] = 4[1287-10] = 4(1277) = 5108 \text{ hands.}$$

C. There are C(52,13) possible bridge hands, and only 4 of them contain all cards of the same suit. Thus, the answer is $4/C(52,13) = 1/\ 158753389900 = 6.30 \times 10^{-12}$.

D. The probability of winning on the first roll (with a 7 or an 11): 6/36 + 2/36 = 8/36 = 2/9.

The probability of rolling a 4 or 10 in the first roll: 3/36+3/36 = 1/6. The probability of winning after rolling a 4 or 10 (rolling a 4 before a 7): 3/(3+6) = 1/3. Thus, the probability of winning AND rolling a 4 or 10 on the first roll is 1/6(1/3) = 1/18.

Similarly, the probability of winning AND rolling a 5 or 9 is 8/36[4/10] = 4/45.

Similarly, the probability of winning AND rolling a 6 or 8 is 10/36[5/11] = 25/198.

The above list exhausts all the possible ways to win. We conclude that the probability of winning is 2/9 + 1/18 + 4/45 + 25/198 = 5/18 + 4/45 + 25/198

$$= 11/30 + 25/198 = 244/495 = .49293$$