A.

8.10 The sample space of all possible sequences is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

a. For each sequence in the sample space, count the number of tails. Then count how many outcomes there are for each possible number of tails. Divide by 8 because all outcomes in the sample space are equally likely. The distribution (*pdf*) is:

k	0	1	2	3
$P(X=k)$	1/8	3/8	3/8	1/8

b. For each sequence in the sample space, take the difference (Y) between the number of heads and the number of tails. For instance, for HHT, $Y = 2$ (heads) $-$ 1 (tails) = 1. List all possible differences, count how many sequences in the sample space have each possible value, then divide by 8 to get the probability of that value. The distribution (*pdf*) is:

k	-3	-1	1	3
$P(Y=k)$	1/8	3/8	3/8	1/8

c. The sum of heads and tails is 3 for every sequence which means $P(Z=3) = 1$ so the distribution (*pdf*) is:

k	3
$P(Z=k)$	1

8.21 The distribution of the sum is given in Example 8.5. The expected value is

$$\mu = E(X) = 2\times\frac{1}{36}+3\times\frac{2}{36}+4\times\frac{3}{36}+\ \ldots\ +10\times\frac{3}{36}+11\times\frac{2}{36}+12\times\frac{1}{36}=\frac{252}{36}=7$$

8.22 $\sigma = 2.415$. The formula is $\sigma = \sqrt{\sum (x-\mu)^2 p(x)}$ where the sum is over all values of x. Here, the mean is $\mu = 7$ (see Exercise 8.10). Details of the calculations are:

X	2	3	4	5	6	7	8	9	10	11	12
$x-\mu$	-5	-4	-3	-2	-1	0	1	2	3	4	5
$(x-\mu)^2$	25	16	9	4	1	0	1	4	9	16	25
$p(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$\sigma = \sqrt{25\frac{1}{36}+16\frac{2}{36}+9\frac{3}{36}+4\frac{4}{36}+1\frac{5}{36}+0\frac{6}{36}+1\frac{5}{36}+4\frac{4}{36}+9\frac{3}{36}+16\frac{2}{36}+25\frac{1}{36}}$$
$$= \sqrt{210/36} = 2.415.$$

8.36 Use a binomial distribution with $n = 10$, $p = .5$. Let X = number of games won by human. The answers can be found using any of the methods discussed in Section 8.4, including the use of Minitab or Excel.

a. $P(X = 5) = .2461$.

b. $P(X = 3) = .1172$. If computer wins 7 games, then human wins 3 games.

c. $P(X \geq 7) = .1719$, calculated as $P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10) =$ $.1172 + .0439 + .0098 + .0010 = .1719$. Equivalently, $P(X \geq 7) = 1 - P(X \leq 6)$ $= 1 - .8281$.

8.46 **a.** X is a uniform random variable (and it is continuous).

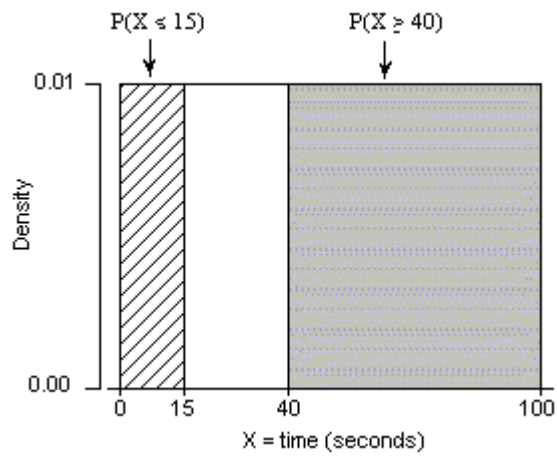
b. X ranges from 0 to 100 and the area under any density curve is 1, so $f(x) = 1/100 = .01$ for all x between 0 and 100. This creates a rectangle (with area=1) similar to Figure 8.2. *Note*: $f(x) = 0$ for any x not between 0 and 100.

c. $P(X \leq 15$ seconds) is the area of the rectangle from 0 to 15 seconds. The interval width is 15 and the height is 1/100, so the answer is (15)(1/100) = .15.

d. $P(X \geq 40$ seconds) is the area of the rectangle between 40 and 100. The interval width is 60 and the height is 1/100 so the answer is (60)(1/100)= .60.

e.

Figure for Exercise 8.46e



f. The expected value or mean is 50. The distribution is symmetric, so the mean equals the median. For a uniform random variable, the median is at the middle of the range of possible values.

8.78 Solve this problem using a binomial distribution with n=10 and p=0.1. $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - .9298 = .0702$. *Note:* If using the binomial formula to find $P(X \leq 2)$ by hand, calculate $P(X=0)+P(X=1)+P(X=2)$ which will be .3487+.3874+.1937=.9298.

B. Suppose that a random variable X has cdf given by the function F(k) = sqrt(k) for values of k between 0 and 1. (And F(k)=0 for negative k, and F(k)=1 for k greater than 1.)
Find the expectation and standard deviation of X. Show all your work.

CDF is F(k) = $\begin{cases} \sqrt{k} & 0 \le k \le 1 \\ 0 & k < 0 \\ 1 & k > 1 \end{cases}$ To find the expectation of X, E(X), we need to define the pdf of X, f(k).

f(k) is the derivative of F(k). So f(k) = $\begin{cases} \frac{1}{2}k^{-1/2} & 0 \le k \le 1 \\ 0 & \text{otherwise} \end{cases}$

Now we can go ahead and find E(X). E(X) = $\int_0^1 \frac{1}{2}k^{-1/2} * k \text{ dk} = \int_0^1 \frac{1}{2}k^{1/2} \text{ dk} = \frac{1}{3}k^{3/2}\Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$

To find the standard deviation of X, we need to find E(X^2) first.

E(X^2) = $\int_0^1 \frac{1}{2}k^{-1/2} * k^2 \text{ dk} = \int_0^1 \frac{1}{2}k^{3/2} \text{ dk} = \frac{1}{5}k^{5/2}\Big|_0^1 = \frac{1}{5} - 0 = \frac{1}{5}$

The standard deviation of X is the square root of its variance. Var(X) = E(X^2) - (E(X))^2 = $\frac{1}{5} - \left(\frac{1}{3}\right)^2 = \frac{4}{45}$

The standard devaition of X= $\sqrt{\frac{4}{45}} \approx 0.2981$

C. Suppose that a continuous random variable Y has pdf that is equal to f(k) = c(k)(k-1) for some constant c and for all k in the interval (1,3). Assume that the pdf is zero outside the interval (1,3). First, find the value of c. Then, calculate E(Y). Show all your work.

f(k) = $\begin{cases} c(k)(k-1) & 1 \le k \le 3 \\ 0 & \text{otherwise} \end{cases}$

We can solve for c since we know that every pdf has to integrate to 1.

So 1 = $\int_1^3 \text{c k (k -1) dk} = c\int_1^3 (\text{k}^2 - \text{k}) \text{ dk} = c\left(\frac{\text{k}^3}{3} - \frac{\text{k}^2}{2}\Big|_1^3\right) = c\left(\frac{26}{3} - \frac{8}{2}\right) = c\frac{14}{3} \rightarrow c = \frac{3}{14}$

E(Y) = $\frac{3}{14}\int_1^3 \text{k} * (\text{k}^2 - \text{k}) \text{ dk} = \frac{3}{14}\int_1^3 (\text{k}^3 - \text{k}^2) \text{ dk} = \frac{3}{14}\left(\frac{\text{k}^4}{4} - \frac{\text{k}^3}{3}\Big|_1^3\right) = \frac{3}{14}\left(\frac{80}{4} - \frac{26}{3}\right) = \frac{17}{7} \approx 2.4286$