

Solutions for Jan. 27 homework in STAT 220.

Most of these answers are taken from the solutions manual for the Utts and Heckard textbook.

Turn On Your Mind:

9.1 (p.295) The key thing to notice here is that the histogram describes the means of different samples of 1900 people, not the individual values of hours of sleep. It may be intuitive that generally the mean of a sample of 1900 people will be a more accurate estimate of the population mean than the mean of a sample of 190. So, with samples of 1900 people the sample means should cluster more tightly around the true population mean and the histogram would be less variable. Put another way, different samples of 1900 people will have means closer to each other than different sample of 190 people would. The theory for this question will be covered in Section 9.3.

9.2 (p.298) In scenario 1, a "success" is an individual response in favor of the candidate. In scenario 2, a "success" occurs when a television set is tuned to the program of interest. In scenario 3, a "success" is an individual preference for the new product. In scenario 4, a "success" is a correct guess at the symbol on the hidden card. To create a new scenario, imagine either a survey in which the objective is to estimate the proportion with a particular trait or opinion (for instance, in favor of a new law), or a random process that can be repeated many times (like playing a casino game or flipping a coin).

9.3 (p.299) By the Empirical Rule, 95% of the values will be within 2 standard deviations of the mean. In Example 9.2, the mean is .40 and the standard deviation is .01 so the range for 95% of the possible values of the sample proportion is $.40 \pm 2(.01)$, or .38 to .42. With $n=600$ rather than $n=2400$, this range will be wider because the standard deviation will be greater. The intuition is that sample proportions for different samples of $n=600$ will be less likely to be close to the true proportion (.40) than the sample proportions for samples of $n=2400$. If $n=600$ is used in the formula for standard deviation on p. 299, the answer is $s.d.(\hat{p}) = .02$.

9.5 (p.306) The range of individual weight losses in the sample would be likely to increase somewhat if the sample size were increased fourfold, but not by much. With a much larger sample, there's more chance of selecting individuals whose weight-losses are at the extremes. But as noted on pages 301-302, 95% of all weight losses are between about -2 and $+18$, and almost all weight losses are between -7 and $+23$. So regardless of the sample size, the minimum weight loss is likely to be no smaller than -7 and the maximum no more than $+23$ pounds. The range of individual observations does not change by an order of magnitude like the range of possible sample means does.

Exercises

- 9.18** a. Mean = .70; s.d. (\hat{p}) = $\sqrt{\frac{.70(1-.70)}{200}} = .0324$.
 b. $.70 \pm (3 \times .0324)$, or .6028 to .7972.
 c. $\hat{p} = 120/200 = .60$. This is a statistic.
 d. The value .60 is slightly below the interval of possible sample proportions for 99.7% of all random samples of 200 from a population where $p = .70$. In other words, the sample proportion is unusually low if the true proportion were .70. Maybe the population value is actually less than .70.

- 9.28** a. Mean = 7.05 hours.
 b. $s.d.(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{1.75}{\sqrt{190}} = 0.127$ hours.
 c. .6.923 and 7.177, calculated as $7.05 \pm .127$.
 d. 6.796 and 7.304, calculated as $7.05 \pm (2 \times .127)$.

- 9.37** a.

Sample	H	Sample	H
1,2	2	2,4	4
1,3	3	2,5	5
1,4	4	3,4	4
1,5	5	3,5	5
2,3	3	4,5	5

- b.

H	2	3	4	5
Probability	1/10	2/10	3/10	4/10

- c. 5

- 9.46** Answer = 1. Mean = $p = .10$; s.d. (\hat{p}) = $\sqrt{\frac{.10(1-.10)}{100}} = .03$; $z = \frac{.13-.10}{.03} = 1$.

- 9.49** The population proportion p must also be known. The relevant formula is

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

- 9.56** The symbol s represents the standard deviation of a sample. The symbol σ represents the standard deviation of a population.

9.80 a. With a sample of $n=1600$ and a true proportion of $p=.56$, the standard

$$\text{deviation is } s.d.(\hat{p}) = \sqrt{\frac{.56(1-.56)}{1600}} = .0124$$

About 68% of all potential sample proportions are in the range $.56 \pm .0124$,
About 95% of all potential sample proportions are in the range $.56 \pm (2 \times .0124)$ or
.535 to .585.

Almost always, the sample proportion will be in the range $.56 \pm (3 \times .0124)$ or
.523 to .597.

b. Based on part (a), .61 does not seem like a reasonable sample proportion because it is outside the range given for the sample proportions that should occur almost always. The standardized score for .61, if the true proportion is .56, is $(.61 - .56) / .0124 = 4.03$.

c. For $n=400$, $\sqrt{\frac{.56(1-.56)}{400}} = .025$. The reported percentage of 61% then corresponds to a standardized score of $(.61 - .56) / .025 = 2$. Only 2.5% of the potential sample proportions would have standardized scores larger than 2 if the true proportion is .56. If everybody told the truth, the sample result is unusual, but not nearly as unusual as with the sample size of 1600. This example illustrates the role of the sample size in assessing whether a sample result is inconsistent with a potential population value.

9.83 Some R code is included below. The dataset is loaded using the following command:

```
>read.table("http://www.stat.psu.edu/~dhunter/220/files/datasets/ascii/GSS-93.txt",header=TRUE, na.strings="*")
```

a.

```
> table(gss93[, "gunlaw"])
```

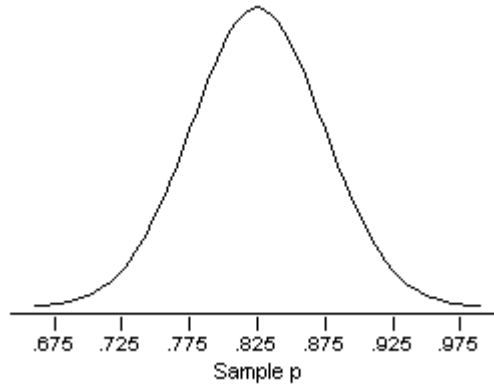
```
  Favor Oppose  
    870    185
```

Thus, $p\text{-hat} = 870/(870+185) = .825$

b. The sampling distribution of the sample proportion is approximately a normal curve with mean $p = .825$ and standard deviation

$$s.d.(\hat{p}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.825(1-.825)}{60}} = .049$$

Figure for Exercise 9.83b



c and d. The answer will differ for each student and the method used will depend upon available software. The histogram will look roughly like the figure drawn for part (b). The figure shown here is the result of one (random) simulation. Note: This figure was not produced using R, but a histogram produced using R would look quite similar in shape.

Figure for Exercise 9.83c

