Let X be a binomial (n, p) random variable. Recall:

- If there are x successes, there must also be $n - x$ failures.
- By independence, any single outcome with x success and $n - x$ failures has probability $p^x(1 - p)^{n-x}$.
- There are $\binom{n}{x}$ such outcomes.

Therefore, the PMF is $f_X(x) = \binom{n}{x} p^x(1 - p)^{n-x}$.

In the special case of $n = 1$, we also call such a random variable a *Bernoulli* random variable.

A company prices hurricane insurance using the following assumptions:

- In any calendar year, there can be at most one hurricane.
- In any calendar year, the probability of a hurricane is 0.05.
- The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

Example 2.11 Probability that an integrated circuit is defective is p. Let X be the number of tests up to and including the first test that discovers a defective one. The PMF of X is given by $f_X(x) = p(1 - p)^{x-1}$ for $y = 1, 2, 3, \ldots$

Goal: Counting "blips" during a fixed interval

- What's a blip? (Warning: Nontechnical and nonstandard word! Avoids overuse of words like "event")
- A blip is a randomly timed happening of some sort: A person joining a line at the bank; a typographical error in a book; a birth in a particular town; number of hits at a Web site in any time interval.
- See examples on pages 60-61.

Let $\lambda > 0$. An approximate Poisson (λ) process assumes:

- Counts of blips in non-overlapping intervals are independent.
- For small h, P(one blip during an interval of length h) is λh plus something insignificant.
- For small h, P(more than one blip during an interval of length h) is something insignificant.

All of this can be made mathematically precise!

So if X is a Poisson (λ) random variable where $\lambda > 0$, its PMF is exactly

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x \in \{0, 1, 2, \ldots\}$$

By the way, λ does not have to be an integer, but X does.

The cumulative distribution function or simply 'distribution function' of a random variable is the function F_X given by

$$F_X(x) = P(X \leq x).$$

Theorem 2.2:
CDF is non-decreasing; $F_X(x)$ approaches 0 as x approaches $-\infty$; $F_X(x)$ approaches 1 as x approaches ∞.
The c.d.f. is the easiest way to understand the difference between chapter 2 ("discrete") and chapter 3 ("continuous").

Students will be able to:

- state and use the PMF of a binomial (n, p) r.v.;
- state and use the PMF of a Geometric r.v.;
- state and use the PMF of a Poisson r.v.
- derive PMF and CDF from each other.