Oct.7 Desired outcomes from last class

Students will be able to:

- derive a marginal PMF from a joint PMF;
- use a joint PMF to check independence.
- give the joint mass function for a trinomial distribution.

Oct.7 4.4, 4.5, 4.10 Joint PDF, Marginal PDF & Independent R.V.

Big shift now:

- Discrete → Continuous
- Joint PMF → Joint PDF (or just "joint density")
- Summation → Integration
- $\sum\sum_{(x,y)\in S} f(x,y) = 1 \quad \rightarrow \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$
- $P[(X,Y)\in A] = \sum\sum_{(x,y)\in A} f(x,y) \quad \rightarrow$ ____*fill in the blank*____

Example: Let X and Y have the joint density

$$f(x,y) = cxy \quad \text{for } 0 < x < 1,\ 0 < y < 2.$$

- What is c?
- What is $P(X^2 + Y^2 \le 1)$?
- Asked another way: What is $P[(X,Y)\in A]$ where $A = \{(x,y) : 0 < x^2 + y^2 < 1\}$?

To find the marginal density for one variable from the joint density, "integrate out the other variable."

- Thus, $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$.
- Similarly, $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$.

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Example: Let $f(x, y) = \frac{6}{5}(x^2 + y)$ for $0 < x < 1$, $0 < y < 1$.

- Find $f_X(x)$.
- Find $f_Y(y)$.

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If X and Y have joint density $f(x, y)$,
then X and Y are independent if and only if

$$f(x, y) = f_X(x) f_Y(y) \quad \text{for all values of } x \text{ and } y.$$

- $f(x, y) = \frac{6}{5}(x^2 + y)$ for $0 < x < 1$, $0 < y < 1$. Independent? NO
- $f(x, y) = 6x^2y$ for $0 < x < 1$, $0 < y < 1$. Independent? YES

Oct.7 Desired Outcomes

Students will be able to:

- understand a joint density of two continuous random variables;
- set up and perform double integrals to find joint probabilities;
- derive a marginal density from a joint density.