Students will be able to:

- derive mean and variance of sum of the random variables
- find distributions of sums of independent r.v.

Sum of two independent Uniform r.v. on $(0, 1)$

$$f_{X+Y}(a) = \int_0^1 f_X(a-y)\,dy = \begin{cases} a & 0 \le a \le 1 \\ 2-a & 1 < a < 2 \\ 0 & \text{otherwise} \end{cases}$$

- Sum of two independent exponentials with the same λ has a gamma distribution
- Sum of two independent gammas with the same λ has a gamma distribution
- Sum of two independent binomials with the same p has a binomial distribution
- Sum of two independent Poissons has a Poisson distribution
- Sum of two independent normals has a normal distributions where means and variances add up.

Example Quiz 6.2 on page 248:
Let X and Y be two independent exponential random variables with epected values $E(X) = 1/3$ and $E(Y) = 1/2$. Find the PDF of $X + Y$.

Sum of two independent binomials with the same p has a binomial distribution

$$
\begin{aligned}
P(X+Y=k) &= \sum_{i=0}^{n} P(X=i, Y=k-i) = \sum_{i=0}^{n} P(X=i)P(Y=k-i) \\
&= \sum_{i=\max(0,k-m)}^{\min(n,k)} \binom{n}{i} p^i (1-p)^{n-i} \binom{m}{k-i} p^{k-i} (1-p)^{m-k+i} \\
&= \binom{n+m}{k} p^k (1-p)^{m+n-k}
\end{aligned}
$$

Sum of two independent Poissons has a Poisson distribution

$$
\begin{aligned}
P(X+Y=n) &= \sum_{k=0}^{n} P(X=k, Y=n-k) = \sum_{k=0}^{n} P(X=k)P(Y=n-k) \\
&= \sum_{k=0}^{n} \frac{\lambda^k}{k!} e^{-\lambda} \frac{\nu^{n-k}}{(n-k)!} e^{-\nu} = \frac{(\lambda+\nu)^n}{n!} e^{-(\lambda+\nu)}
\end{aligned}
$$

Students will be able to:

- identify sums of certain independent random variables.