Nov. 7 Announcements

Exam 2 on Friday Nov. 11 covers

- Section 3.7
- Chapters 4 and 5 (excluding sections 4.12, 5.8)
- sections 6.1, 6.2, 6.3, 6.4, 6.5.

Nov. 7 Desired outcomes from last class

Students will be able to:

- find MGF of sums of independent r.v.;
- derive bivariate normal distribution.

Nov. 7 5.7 Gaussian Random Vectors

Example:

- Let X_1 and X_2 be independent standard normal random variables.
- For some $0 < |\rho| < 1$, let $(Y_1, Y_2) = (X_1, \rho X_1 + \sqrt{1-\rho^2} X_2)$.
- Joint density of (Y_1, Y_2) is bivariate normal with correlation coefficient ρ.

Nov. 7 Joint Probability Distribution of Functions of Random Variables

Suppose that

- (X_1, X_2) has joint density $f(x_1, x_2)$.
- (Y_1, Y_2) is some function of (X_1, X_2).
- The inverse function $[x_1(y_1, y_2), x_2(y_1, y_2)]$ is single-valued.

Then $f_{Y_1,Y_2}(y_1, y_2) = \left|\text{Det}\begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{pmatrix}\right|^{-1} f_{X_1,X_2}[x_1(y_1, y_2), x_2(y_1, y_2)]$ is the p.d.f.

Nov. 7 6.5 Random sum of independent r.v.

Example: Suppose that the number N of customers entering Walmart on a given day has Poisson distribution with mean 50. Suppose further that the amounts spent by these customers in the store are independent random variables having a common exponential distribution with mean \$100. Finally, the amount of money spent by a customer is also independent of the total number of customers who enter the store. Find the MGF of the total amount of money taken by the store on a given day? Find the expected amount of money taken in by the store?

Nov. 7 6.5 Random sum of independent r.v.

Suppose X_i denote the amount of money spent by the i-th customer. Thus $S = \sum_{i \le N} X_i$ is the total amount of money taken by the store on a given day. Let M denote the MGF of X_1.

$$\begin{aligned} M_S(t) =& E(e^{tS}) = E(E(e^{tS}|N)) = E((M(t))^N) \\ =& \sum_{i=0}^{\infty} (M(t))^n P(N = n) = \sum_{i=0}^{\infty} (M(t)50)^n \frac{1}{n!} e^{-50} = e^{50(M(t)-1)} \end{aligned}$$

$M_S'(0) = 50M'(0) = 50 \times 100 = 5000$

Nov. 7 Desired Outcomes

Students will be able to:

- use the change-of-variables formula to derive bivariate normal distribution
- derive MGF of random sum of independent r.v.