Students will be able to:

- Describe Stochastic process;
- work with Poisson process;
- recognize and apply Brownian Motion;
- derive expectation function, correlation and covariance functions.

Example 10.20 on page 372: Input is an i.i.d. sequence $\ldots, X_{-1}, X_0, X_1, \ldots$ with variance 1 and $E(X_i) = 0$.
Output sequence $Yn = X_n + X_{n-1}$

$$C_Y(m,k) = R_Y(m,k) = \begin{cases} 2 & k = 0 \\ 1 & k = -1, 1 \\ 0 & \text{otherwise} \end{cases}$$

$X(t)$ is stationary if for all s, $t_1, \ldots, t_r$, $r \geq 1$, the joint distribution of $(X(t_1), \ldots, X(t_r))$ and $(X(t_1 + s), \ldots, X(t_r + s))$ are identical. In this case for all s, t, $\mu_X(t) = \mu_X, \quad and \quad R_X(t,s) = R_X(0,s)$
Recall $R_X(t,s) = E[X(t)X(t+s)]$. Marginal distributions are identical.
Similar definition for discrete time processes.

$X_1, X_2, \ldots$ is i.i.d., then it is stationary.
$X(t)$ stationary. Check if the following are stationary

- For $a > 0$ and b, $Y(t) = aX(t) + b$
- $Y_n = X(n/2)$
- A is independent of $X(t_1), \ldots, X(t_k)$ for all $k \geq 1$, and $t_1, \ldots, t_k$. $Y(t) = AX(t)$. What about $Y(t) = A + X(t)$?

$X(t)$ is a wide sense stationary stochastic process if
$\mu_X(t) = \mu_X$, $R_X(t,s) = R_X(0,s)$ for all s, t.

Nov. 30 10.10 Wide Sense Stationary Stochastic Processes

Example: $X(t)$is a wide sense stationary process and $Y(t) = X(-t)$.
Express R_Y in terms of R_X.
Is Y a wide sense stationary process?

- $X(t)$ is a Gaussian Process if $(X(t_1), \ldots, X(t_r))$ has multivariate normal distribution for all $t_1, \ldots, t_r$, $r \geq 1$.
- Brownian Motion is a Gaussian Process.
- Important result. If $X(t)$ is a wide sense stationary Gaussian Process, than it is a stationary Gaussian process.
- White Gaussian Noise W: $W(t)$ are i.i.d. Gaussian random variables.

Nov. 30 10.12 Gaussian Processes

$X(t)$ is stationary Gaussian process with $\mu_X(t) = 0$ and autocorrelation $R_X(t) = 2^{-|t|}$. What is the joint PDF of $X(t), X(t+1))$.

Students will be able to:

- Describe Stochastic process;
- work with Poisson process;
- recognize apply Brownian Motion;
- derive expectation function, correlation and covariance functions;
- understand the difference between the stationary and stationary in wide sense processes;
- define Gaussian Processes