Students will be able to:

- identify sums of certain independent random variables.

Clever idea: Try differentiating $E\left[e^{tX}\right]$ with respect to t:

$$\frac{\partial^r}{\partial t^r} E\left[e^{tX}\right] = \frac{\partial^r}{\partial t^r} \sum_{x \in S} e^{tx} f_X(x) = \sum_{x \in S} x^r e^{tx} f_X(x)$$

Setting $t = 0$, the last expression is just $E(X^r)$.

The *moment generating function* for a random variable X is

$$M_X(t) = E\left(e^{tX}\right).$$

- As with the PDF $f_X(x)$, sometimes we omit the subscript.
- $M'(0) = E(X)$ and $M''(0) = E(X^2)$ and $M'''(0) = E(X^3)$ and...

- $Y = a + bX$, $M_Y(t) = e^{ta} M_X(bt)$
- MGF of standard normal: $M(t) = e^{\frac{1}{2}t^2}$
- MGF of normal with mean μ and Variance σ^2: $M(t) = e^{t\mu + \frac{1}{2}\sigma^2 t^2}$

Moment generating functions

- Pretty good news: Sometimes useful for calculating moments.
- Really good news: Uniquely determine the probability distribution.
- Bad news: Not all random variables have a well-defined MGF

Example: Find the MGF of standard normal variable.
Find the mean and variance of a random variable whose MGF is

$$M(t) = e^{3t+4t^2}.$$

Example: Suppose X takes only two values, 0 or 1. (*Bernoulli* r.v.)

- Let $p = P(X = 1)$ so $P(X = 0) = 1 - p$.
- PMF can be written $f(x) = p^x(1 - p)^{1-x}$, for $x = 0, 1$.
- What is the MGF?

Students will be able to:

- derive the moment generating function of a r.v.;
- find $E(X^r)$ for an arbitrary r using the MGF of X.