

6.5.1 (b) By Theorem 6.12,  $\Phi_V(s) = \Phi_K(\ln \Phi_R(s))$

$$\Phi_R(s) = \frac{1}{1-s}, \quad \Phi_K(s) = (1-q)e^s / \{1-qe^s\}$$

$$\Rightarrow \Phi_V(s) = (1-q)\lambda / \{1-s-q\lambda\}$$

$\Rightarrow V \sim \text{Exponential}((1-q)\lambda)$

$\Rightarrow$  pdf of  $V$  is

$$f_V(v) = (1-q)\lambda \exp\{-\lambda(1-q)v\} \quad v \geq 0$$

6.5.6.

(a)  $K+1 \sim \text{Geometric}(q) \Rightarrow \Phi_K(s) \cdot e^s = \frac{qe^s}{1-(1-q)e^s}$

$$\Rightarrow \Phi_K(s) = q / \{1-(1-q)e^s\}$$

(b)  $\Phi_R(s) = \exp\{s + s^2/2\}$

Then  $\Phi_V(s) = \Phi_K(\ln \Phi_R(s)) = q / \{1-(1-q)\exp\{s + \frac{s^2}{2}\}\}$

$$(c) \quad ER = \frac{d}{ds} \Phi_V(s) \Big|_{s=0} = \frac{1-q}{q} \Rightarrow \text{Var}R = ER^2 - (ER)^2 = \frac{1-q^2}{q^2}$$
$$ER^2 = \frac{d^2}{ds^2} \Phi_V(s) \Big|_{s=0} = \frac{2(1-q)}{q^2}$$

6.6.3 (a) \$36 is the cost of 315 minutes, and  $EX = 2.5$ ,  $\text{Var}X = 2.5^2 = 6.25$

By C.L.T.  $P(Y > 315) = P(Z > \sqrt{120} \left( \frac{315}{120} - 2.5 \right) / 2.5)$   
 $= \Phi\left(\frac{15}{\sqrt{120}}\right) = 0.2919$

(b) Let  $W_i = \lceil X_i \rceil \Rightarrow W_i$  is geometric with  $p = 1 - e^{-1} = 0.3297$

$$\Rightarrow E W_i = 3.033, \quad \text{Var} W_i = 6.167$$

$$\Rightarrow P(Y > 315) = P(Z > \sqrt{120} \left( \frac{315}{120} - 3.033 \right) / \sqrt{6.167}) = \Phi(-1.8) = 0.964$$

6.7.3 (a)  $500/0.8 = 625$

(b)  $P(Y \geq 500) = P(Z \geq \sqrt{600} \left( \frac{500}{600} - 0.8 \right) / \sqrt{0.8 \times 0.2}) = \Phi\left(\frac{5}{\sqrt{6}}\right) = 0.10206$

(d)  $\Phi(x) = 0.9 \Rightarrow x = -1.29$

$$\Rightarrow \frac{500 - np}{\sqrt{np(1-p)}} \geq -1.29 \Rightarrow n = 642$$

$$\begin{aligned}
 6.8.1 \quad P[Z \geq c] &\leq \min_{s \geq 0} e^{-sc} \Phi_Z(es) \\
 &= \min_{s \geq 0} \exp\left\{ \frac{s^2}{2} - sc \right\} \\
 &= \min_{s \geq 0} \exp\left\{ \frac{1}{2} (s-c)^2 - \frac{c^2}{2} \right\} \\
 \Rightarrow P(Z \geq c) &\leq \frac{c^2}{2}
 \end{aligned}$$

	$c=1$	$c=2$	$c=3$	$c=4$	$c=5$
Chernoff bound	0.606	0.135	0.011	$3.35 \times 10^{-4}$	$3.73 \times 10^{-6}$
Exact prob.	0.1587	0.0228	0.0013	$3.17 \times 10^{-5}$	$2.87 \times 10^{-7}$