Exam 1 STAT/MATH 418 Fall 2011

1. At a local pizza place you can have either thin crust, hand tossed or pan. The choices for the size are small, medium and large, and the choices for toppings are pepperoni, mushrooms, green peppers, hot peppers, tomatoes, spinach and chicken. In addition, either you can eat in or have it delivered to your room.
   a) How many different possibilities are there if you choose 3 different toppings?

   Solution: 3 choices for crust, 3 choices for size, 7 choices for toppings, and 2 choices for eating place.

$$3 \times \binom{7}{3} \times 3 \times 2 = 630.$$

   b) How many different possibilities are there if you choose 2 toppings and have it delivered?

   Solution:

$$3 \times \binom{7}{2} \times 3 = 189.$$

2. The proportions of motorists at the Exxon gas station in College Heights using regular, extra unleaded, and premium gas are 40%, 35%, and 25%, respectively. The respective proportions of filling their tanks are 30%, 50%, and 60%. If a randomly chosen motorist filled his/her tank, what is the probability that he/she used regular gas.

   Solution: Let R = Regular, E = Extra unleaded, P = Premium, and F = Full tank. By the Bayes formula,

$$\begin{aligned} P(R|F) &= \frac{P(F|R)P(R)}{P(F|R)P(R) + P(F|E)P(E) + P(F|P)P(P)} \\ &= \frac{0.3 \times 0.4}{0.3 \times 0.4 + 0.5 \times 0.35 + 0.6 \times 0.25} = 0.26966 \end{aligned}$$

3. The cumulative distribution function of a random variable X is given by

$$F_X(a) = \begin{cases} 0, & \text{if } a < -1 \\ \frac{1}{2}(1+a), & \text{if } \; -1 \le a < 1 \\ 1, & \text{if } \; a \ge 1. \end{cases}$$

   Find probability density function, mean and variance of X.

   Solution: The probability density function f_X is given by $f_X(a) = F'_X(a) = \frac{1}{2}$ for $-1 < a < 1$, and $f_X(a) = 0$ otherwise. So X is uniformly distributed in $(-1, 1)$.

$$\begin{aligned} E(X) &= \int_{-1}^{1} x \, \frac{1}{2} \, dx = 0 \\ Var(X) &= E(X^2) - (E(X)^2) = \int_{-1}^{1} x^2 \, \frac{1}{2} \, dx = \frac{1}{3}. \end{aligned}$$

4. Suppose E, F and G are events relating to an experiment and $P(G) = 0.45$. If

$$P(E|G) > P(F|G), \text{ and } P(E|G^c) > P(F|G^c),$$

Then either establish $P(F) < P(E)$, or justify your answer if it is false.

Solution: By the properties of conditional probabilities, we have

$$\begin{aligned} P(E) =& P(E \cap G) + P(E \cap G^c) \\ =& P(E|G)P(G) + P(E|G^c)P(G^c) \\ >& P(F|G)P(G) + P(F|G^c)P(G^c) = P(F \cap G) + P(F \cap G^c) = P(F). \end{aligned}$$

5. Consider the function $f(x) = C(x-4)$ if $0 \le x \le 2$ and 0 otherwise.
a) Could f be a probability density function for some C? If so, determine C.

Solution: As $(f(x)/C) = (x-4) < 0$ for $0 \le x \le 2$, f is a non-negative function provided $C < 0$. As $1 = \int_0^2 C(x-4)dx = C(2-8) = -6C$, it is a probability density function if $C = -\frac{1}{6}$.

b) Repeat it if f were given by $f(x) = C(x-4)$ for $0 \le x \le 6$ and 0 otherwise.

Solution: As

$$\frac{f(3)}{C} = -1 < 0 \text{ and } \frac{f(5)}{C} = 1 > 0,$$

there is no value of C for which $f(x)$ is always non-negative. So f cannot be a probability density function.