The algebraic form: \( z = x + iy \) where \( x \) and \( y \) are real numbers, \( i = \sqrt{-1} \); 
\( x \) is called the real part of the complex number \( z \); 
\( y \) is called the imaginary part of the complex number \( z \).

The geometric interpretation in the plane: the point \((x, y)\) or the vector with the origin at \((0, 0)\) and with the end-point \((x, y)\).

The number \( \bar{z} \) conjugate to \( z = x + iy \) is defined as \( \bar{z} = x - iy \).

Addition:
\[
(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2).
\]

Multiplication:
\[
(x_1 + iy_1) \cdot (x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)
\]

Important special case: \( z \cdot \bar{z} = (x + iy)(x - iy) = (xx - y(-y)) + i(x(-y) + xy) = x^2 + y^2 \) (a real number).

Division:
\[
\frac{z_1}{z_2} = \frac{z_1 \cdot \bar{z}_2}{z_2 \cdot \bar{z}_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 + y_2^2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\left(\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}\right), \ z_2 \neq 0.
\]

Properties of the conjugate numbers:
1. \( \bar{\bar{z}} = z \) iff \( z \) is real.
2. \( \bar{z} \) = \( z \)
3. \( \bar{z_1 + z_2} = \bar{z_1} + \bar{z_2} \)
4. \( \bar{z_1 - z_2} = \bar{z_1} - \bar{z_2} \)
5. \( \bar{z_1z_2} = \bar{z_1} \cdot \bar{z_2} \)
6. \( \frac{\bar{z_1}}{\bar{z_2}} = \frac{\bar{z_1}}{\bar{z_2}} \)

The modulus, or the absolute value \( |z| \) of a complex number \( z = x + iy \) is a real number \( |z| \) defined as follows:
\[
|z| = \sqrt{x^2 + y^2}
\]

Main properties of the modulus \( |z| \):
1. The triangle inequality:
\[
|z_1| - |z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|.
\]
2. \( |z| = |\bar{z}| \)
3. \( z \cdot \bar{z} = (x + iy)(x - iy) = x^2 + y^2 = |z|^2 \).
The **argument** $\theta$ of a complex number $z = x + iy$ is defined as the angle (measured in radians) between the vector $(x, y)$ and the axis $x$.

\[
\tan \theta = \frac{y}{x}, \quad \theta = \tan^{-1} \frac{y}{x}.
\]

**Notation:** $\theta = \arg z$.

For each $z$ there are infinitely many values of $\arg z$.

The **principal value** of $\arg z$ (denoted by $\operatorname{Arg} z$) is the value of $\arg z$ such that $-\pi < \arg z \leq \pi$.

\[
\arg z = \operatorname{Arg} z + 2n\pi \quad (n = 0, \pm 1, \pm 2, \ldots).
\]

**Rule:** To multiply complex numbers we multiply their moduli and add their arguments.
SECTION 8

Powers

$z^n$ is defined by the formulas:

$$z^n = z \cdot \ldots \cdot z; z^{-n} = \frac{1}{z^n}, n = 1, 2, \ldots$$

If $z = re^{i\theta}$ then $z^n = r^n e^{i n\theta}$, $n = 0, \pm 1, \ldots$

$|z^n| = |z|^n$

De Moivre’s formula:

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta.$$  

Roots

A number $\omega$ is an $n$-th root of $z$, or a root of order $n$ if $w^n = z$ (notation: $w = \sqrt[n]{z}$ or $z^{\frac{1}{n}}$).

Every complex number $z = re^{i\theta}$ has $n$ roots of order $n$.

The number 1 has the following $n$ roots of order $n$:

$$\omega_0 = 1$$
$$\omega_1 = \exp\left(i \frac{2\pi}{n}\right)$$
$$\ldots$$

$$\omega_k = \exp\left(i \frac{2\pi k}{n}\right)$$
$$\ldots$$

$$\omega_{n-1} = \exp\left(i \frac{2\pi(n-1)}{n}\right)$$

The cyclic property:

$$\omega_k = \omega_{k}^{n}, \omega_1^n = \omega_0.$$

General formula:

The $n$-roots of $z = re^{i\theta}$ are given by the formulas:

$$w_k = \sqrt[n]{r} \exp\left(i \frac{\theta}{n} + i \frac{2k\pi}{n}\right) = \sqrt[n]{r} e^{i\frac{\theta}{n}} \omega_k \quad (k = 0, \ldots, n - 1)$$

The sum of $n$ terms of a geometric progression:

$$1 + z + z^2 + \ldots + z^n = \frac{1 - z^{n+1}}{1 - z} = \frac{z^{n+1} - 1}{z - 1} \quad (z \neq 1).$$