ASSIGNMENT #12

Due Friday, April 27

PROBLEM 8.2-10[8.6-10] (5 pt)

Problem A (2+2=4 pt)

Refer to Problem 8.2-10.
1) What is your conclusion if the significance level \( \alpha = 0.05 \)?
2) The \( p \)-value of the test is 0.021. Use this information to deduce your decisions when \( \alpha = 0.01 \) and when \( \alpha = 0.05 \).

A LITTLE BIT OF REGRESSION

PROBLEM 6.7-2 (a,b) (4+1=5 pt)
PROBLEM 6.7-4 (a-c) (4+2+2=8 pt)

PROBLEM B (2 pt)

Refer to Problem 6.7-4. Use the regression formula to predict the ionic charge (in mV) when the ion concentration is 205 ppm.

Remark. The simple linear regression model used here is not very appropriate in Problem 6.7-4. The parabolic regression model that you will use in Problem 6.8-15 fits here much better and therefore the prediction will be more accurate.

PROBLEM 6.8-8 (5 pt)

Advice. Write the model as

\[
y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 \quad (1)
\]

and then as

\[
y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \quad (2)
\]

where \( x_1 = 1 \) in all measurements; add \( x_1 = 1 \) to all measurements; so the matrix of the measurements (it has 12 rows and 4 columns) becomes
As you can see we denote the four column vectors of this matrix by \( \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{y} \). Now you can write the coefficients \( a_{il} \) of the normal equations using the inner products of the column vectors: 
\[
 a_{il} = \langle \mathbf{x}_i, \mathbf{x}_l \rangle = \sum_{j=1}^{12} x_{ij} x_{lj}; \quad c_i = \langle \mathbf{y}, \mathbf{x}_i \rangle.
\]
Write the normal equations and solve them. The system of the three normal equations is
\[
\begin{align*}
 a_{11} \beta_1 + a_{12} \beta_2 + a_{13} \beta_3 &= c_1 \\
 a_{21} \beta_1 + a_{22} \beta_2 + a_{23} \beta_3 &= c_2 \\
 a_{31} \beta_1 + a_{32} \beta_2 + a_{33} \beta_3 &= c_3.
\end{align*}
\]
Write the regression equation (1) with the coefficients you found.

**PROBLEM A** (3 pt)

Refer to Problem 6.8-8. Find values of \( x_2 \) and \( x_3 \) for which the response \( y = 3 \).

**PROBLEM 6.8-12 (a,b)** (5+3=8 pt)

**PROBLEM B** (3 pt)

Refer to Problem 6.8-12. Derive the normal equations obtained in part (a) in another way: by reduction to the general case of linear regression with four "factors": denote \( x_1 = 1 \) (in all measurements), \( x_2 = x, x_3 = x^2 \) and then consider the model (2). The measurements \( (x_j, y_j) \) for the variables \( (x, y) \) become \( (1, x_j, x_j^2, y_j) \) for the variables \( (x_1, x_2, x_3, y) \) (here \( j = 1, \ldots, n \)).

**PROBLEM 6.8-15 (a-c)** (5+3+3=11 pt)

**Remarks.**

1. The questions (a-c) are stated in Problem 6.7-4.
2. The simple linear regression model used in Problem 6.7-4 appeared to be not appropriate. The parabolic regression model that you used in Problem 6.8-15 fits here much better and therefore the predictions, made using this model, will be more accurate.

**PROBLEM C** (2 pt)

Refer to Problem 6.8-15. Use the new (more precise) regression formula to predict the ionic charge (in mV) when the ion concentration is 205 ppm.