Intermediate Applied Statistics
STAT 460

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Instructor:
Aleksandra (Seša) Slavković
esse@stat.psu.edu

TA:
Wang Yu
wangyu@stat.psu.edu

Linear Contrasts

- So far we have only been comparing two groups at a time. However, we might want to do something more creative such as:
  - Compare one group to the average of two other groups
  - Compare the average of two groups to the average of two other groups
  - Find a rate of increase in Y across levels of a quantitative x
  - See if the first population mean is more than three times larger than the second population mean
- Etc.

Linear contrasts

All of these ideas can each be described as estimating some quantity

\[ \gamma = C_1 \mu_1 + C_2 \mu_2 + \ldots + C_k \mu_k \]

where \( C_1, C_2, \ldots, C_k \) are some set of coefficients. This is called a linear combination of the \( \mu \)'s. (If \( C_1 + C_2 + \ldots + C_k = 0 \) then it is also called a contrast.)

Linear contrasts

- As you might expect, the best estimate for

\[ \gamma = C_1 \bar{Y}_1 + C_2 \bar{Y}_2 + \ldots + C_k \bar{Y}_k \]

is

\[ g = C_1 \bar{Y}_1 + C_2 \bar{Y}_2 + \ldots + C_k \bar{Y}_k \]
Linear contrasts

The standard error (estimated standard deviation) of $g$ is

$$ SE(g) = \sqrt{\frac{MSE \sum \frac{c_i^2}{n_i}}{N-k}} $$

where $s_p$ is pooled over all groups in the ANOVA.

$g$ has a normal distribution if the sampling distributions of the means are normal.

$g/SE(g)$ has a t-distribution with

$$ df_{error} = N-k = n_1-1+n_2-1+...+n_k-1$$

degrees of freedom.

Linear contrasts

- Using this information and the output from an ANOVA table, we can calculate tests and confidence intervals for any contrast we are interested in.
- We don’t have to do any multiple comparison adjustments, as long as our contrasts were planned in advance on theoretical grounds rather than chosen after the experiment by looking at the data.

Linear contrasts

- If we want to do “data snooping” and investigate a contrast after we see the data, we have the problem that the number of possible contrasts we could test is infinite. So we certainly can’t use Bonferroni, for example, since it would set $\alpha' = \frac{\alpha}{\infty} = 0$. And we can’t use Tukey, Dunnett or Hsu’s procedures because they are only for pairwise differences.

Linear Contrasts

- See extra handout on Linear contrasts on the course website
  - Source: Dr. Rathburn